



**Weill Cornell
Medicine**

Differential Deep Learning on Graphs and its Applications

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This Tutorial

- ❑ www.calvinzang.com/DDLG_AAAI_2020.html
- ❑ [AAAI-2020](#)
- ❑ Friday, February 7, 2020, 2:00 PM -6:00 PM
- ❑ Sutton North, Hilton New York Midtown, NYC



This Tutorial

- ✓ **Molecular Graph Generation:** to generate novel molecules with optimized properties
 - Graph generation
 - Graph property prediction
 - Graph optimization
- **Learning Dynamics on Graphs:** to predict temporal change or final states of complex systems
 - Continuous-time network dynamics prediction
 - Structured sequence prediction
 - Node classification/regression
- **Mechanism Discovery:** to find dynamical laws of complex systems
 - Density Estimation vs. Mechanism Discovery
 - Data-driven discovery of differential equations



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Part 1: MoFlow: An Invertible Flow Model for Generating Molecular Graphs

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Background: Drug Discovery



Challenges of Drug Discovery



❑ 1. Lengthy, costly, & with high failure rate

- $1.5 + 3 + 1.5 = 6$ years before clinical trial
- 33% of total cost of medicine development
- Clinical success $\sim 12\%$, poor translation in patients
- How to accelerate the process and reduce the cost and failure rate of such a sequential pipeline?

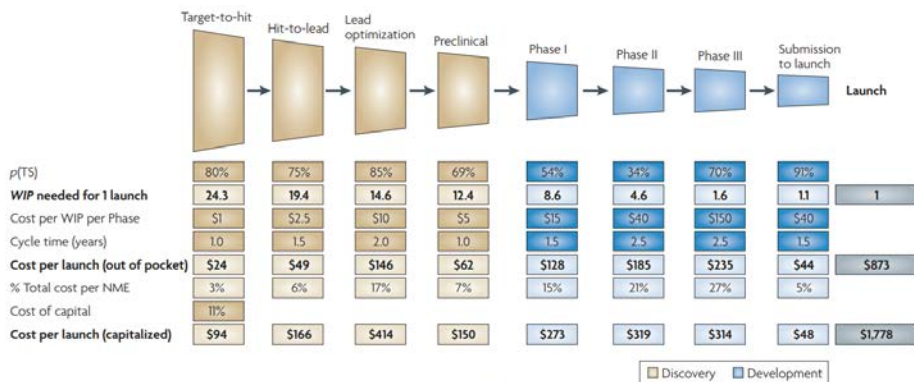
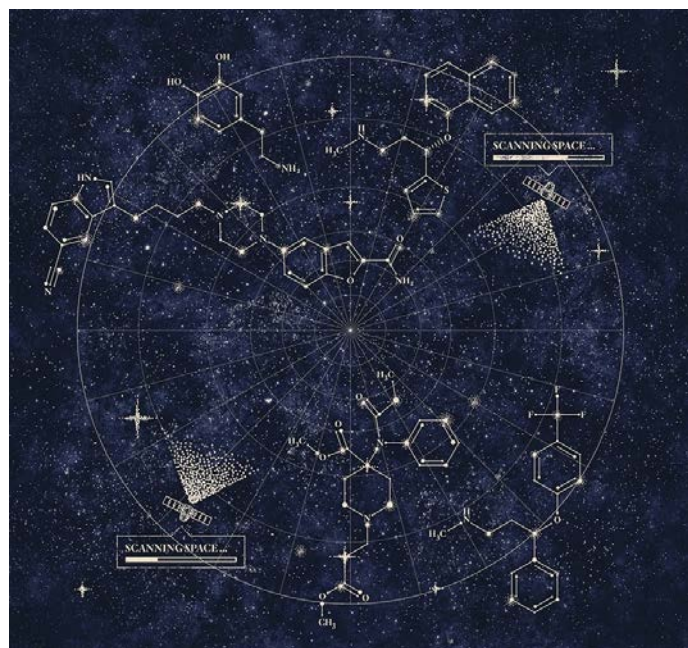
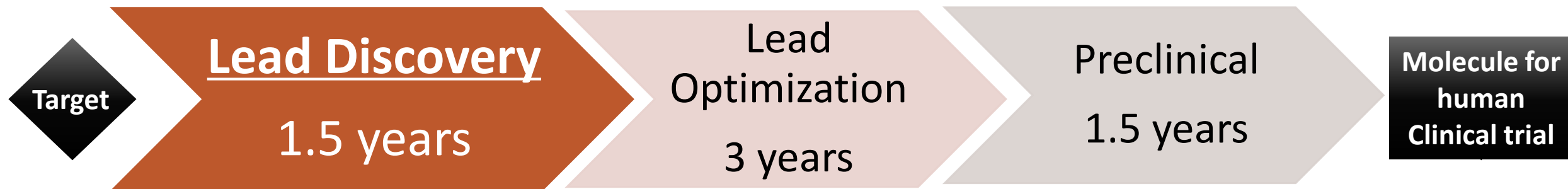


Figure 2 | R&D model yielding costs to successfully discover and develop a single new molecular entity. The model

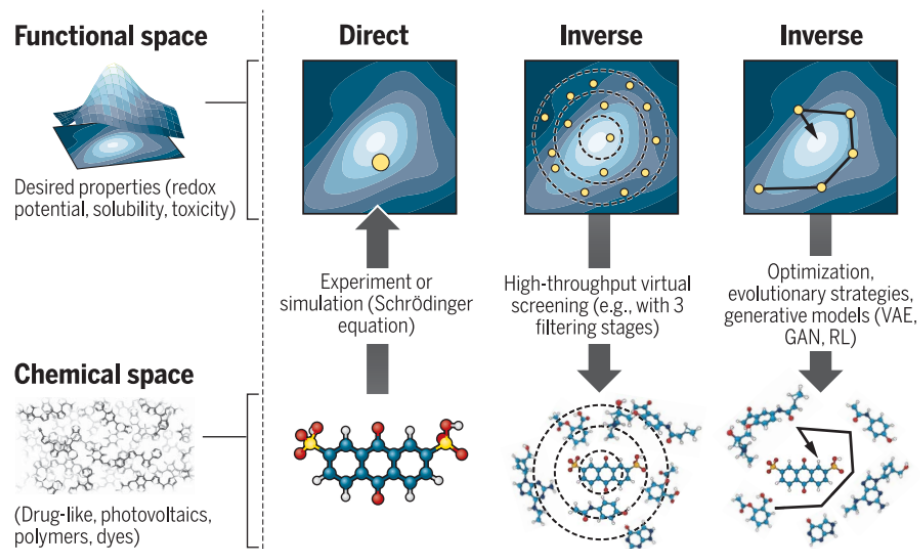
Challenges of Drug Discovery



❑ 2. Big Chemical Data but largely unexplored

- The scale of potential drug-like chemical data: $10^{33} \sim 10^{60}$
- Sampled points in existing chemical database: 10^6
- **How to explore such a big chemical space and generate novel molecule candidates?**

Challenges of Drug Discovery



3. Evaluation and optimization over sequence or graphs

- Discrete Molecule Data: smiles -> sequence, molecular graphs -> graph, etc.
- Evaluation: Mapping from discrete molecules to properties.
- Optimization: Generating novel molecules with optimized properties.
- **How to search for discrete molecules guided by the target property in the chemical space?**

Drug Discovery Driven by Data and AI

Challenges

○ How to accelerate the process and reduce the cost and failure rate of such a sequential pipeline?



○ How to explore such a big chemical space and generate novel molecule candidates?



○ How to search for discrete molecules guided by the target property in the chemical space?



Deep Learning methods driven by data

○ Better than human discovery, hopefully

○ A generative model which approximately represents the large chemical space

○ Search algorithm between the chemical space and property space.

Problem Definition

□ Input:

- $\{M_1, M_2, \dots, M_N\}$: Molecule data samples
- $f(M)$: Some property functions of molecules

□ Method:

- $\hat{M} \sim P_M(M)$: A **generative model (distribution)** of molecules
- Searching molecules in $P_M(M)$ guided by $f(M)$.

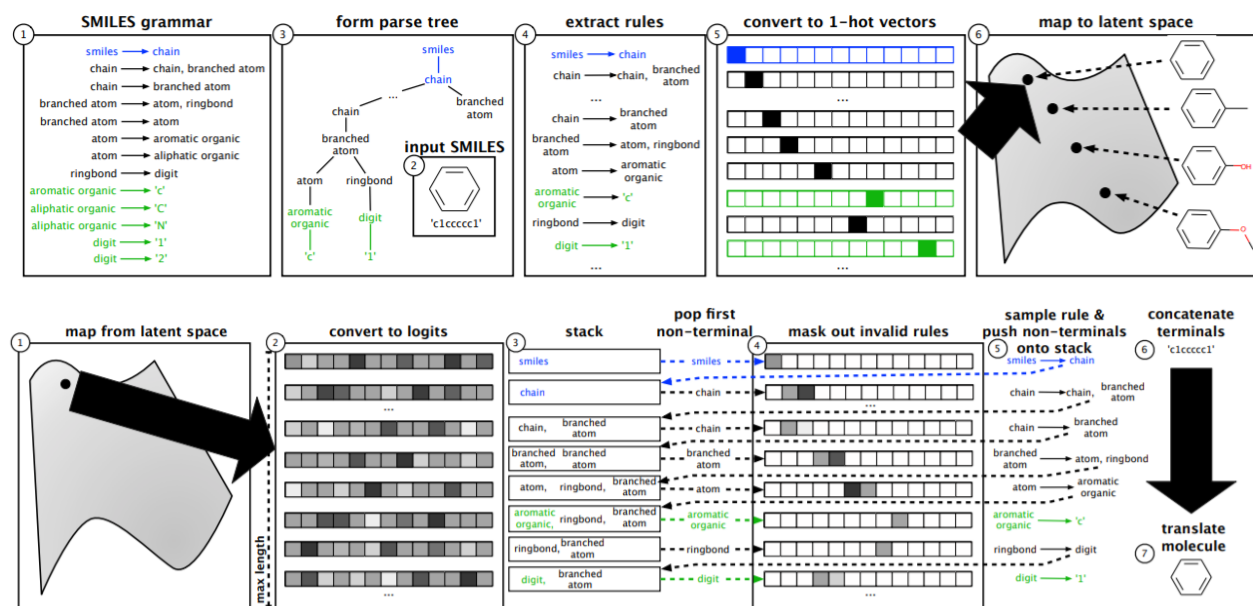
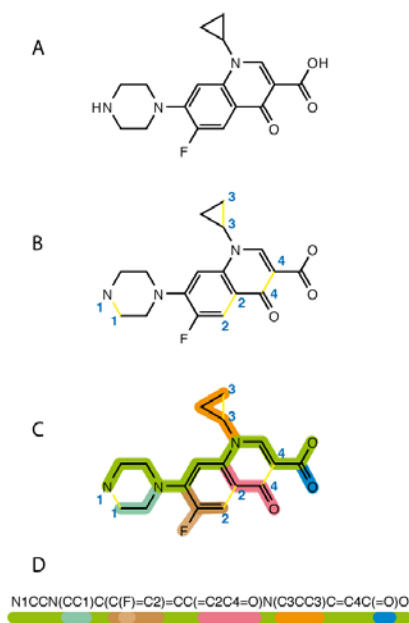
□ Output:

- Novel molecules $\{M_{N+1}, M_{N+2}, \dots\}$ with desired properties.

Related Works

Sequence-based VAE model

- SMILES (Simplified molecular-input line-entry system) string
- Grammar Variational Autoencoders (Grammar-VAE)
- Limitation: Sequences lose structural information



Related Works

Graph-based VAE model

- Structural information of molecules is better kept by graphs
 - ❖ E.g., similarity, chemical validity
- Junction Tree Variational Autoencoder (JT-VAE)
- Limitation
 - ❖ Only for tree-structured molecules.
 - ❖ Ciclosporin: Large circle

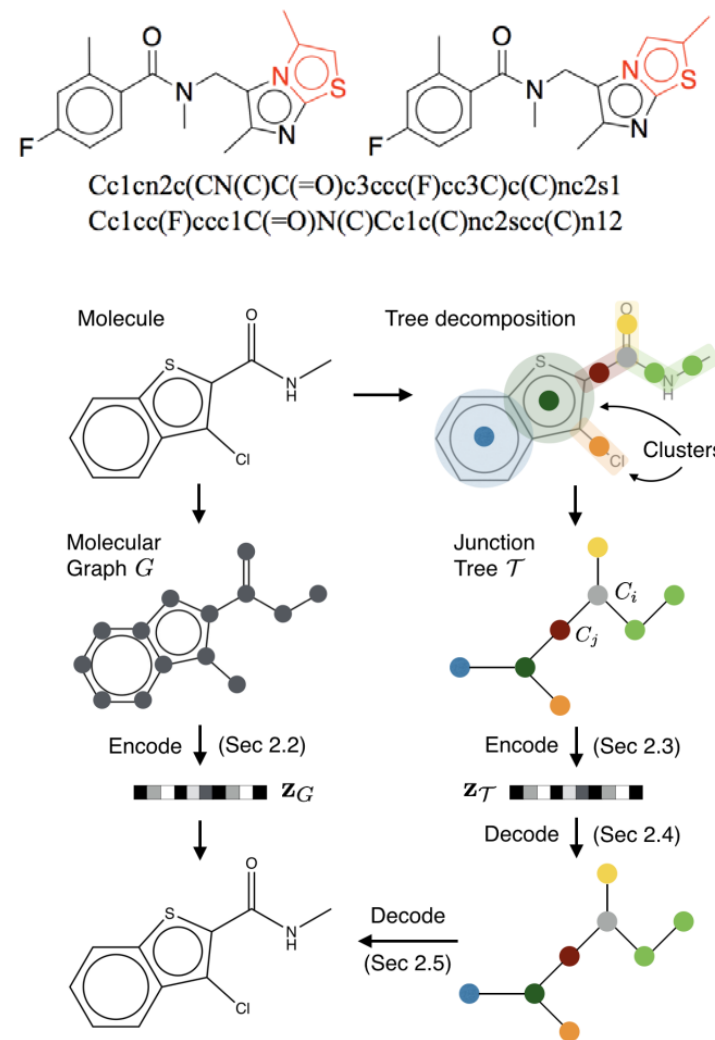
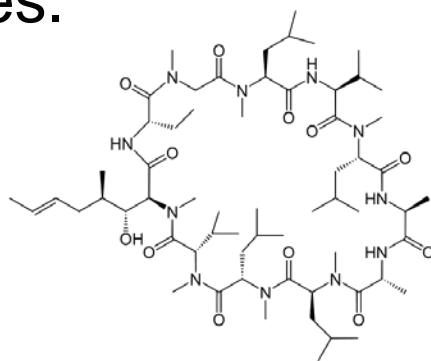


Image from: Jin et al. 2018. [Junction Tree Variational Autoencoder for Molecular Graph Generation](#). *ICML*

<https://en.wikipedia.org/wiki/Ciclosporin>

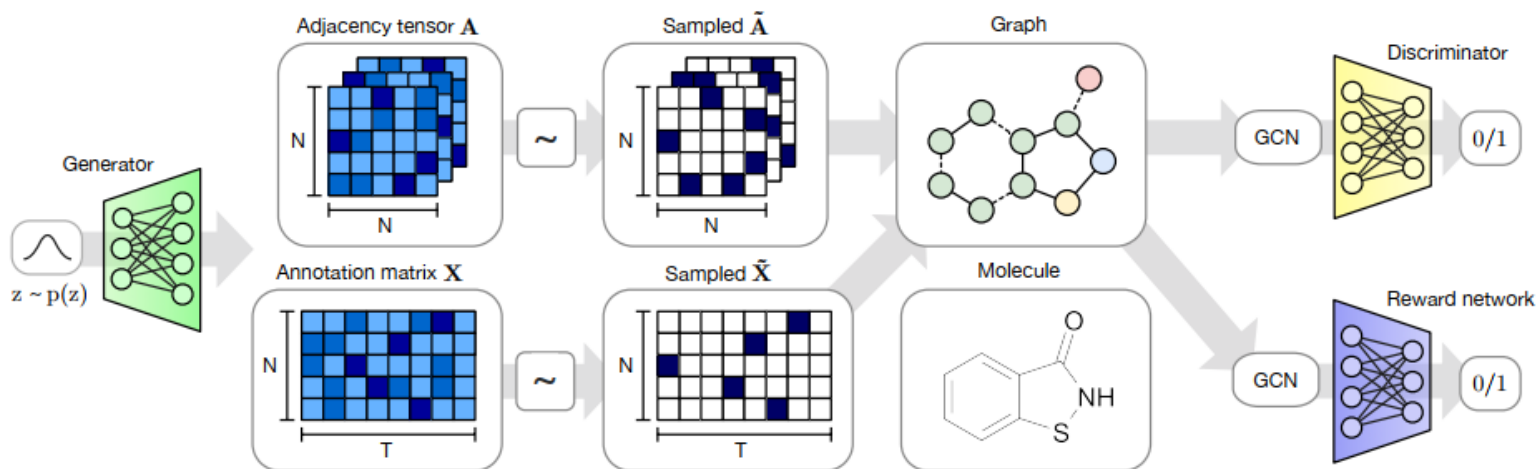
Related Works

GAN-based models

- Molecular Generative adversarial network (MolGAN)

- Limitation

- ❖ No chemical validity guarantee; Mode collapse \rightarrow tend to generate duplicated molecules \rightarrow few novel molecules



Related Works

Autoregressive-based models

- Graph Convolutional Policy Network (GCPN)
- Graph Autoregressive Flow model (GraphAF)
- Reject sampling for validity + Reinforcement Learning for optimization
- Limitations
 - Sequential generation, tend to generate long chains.

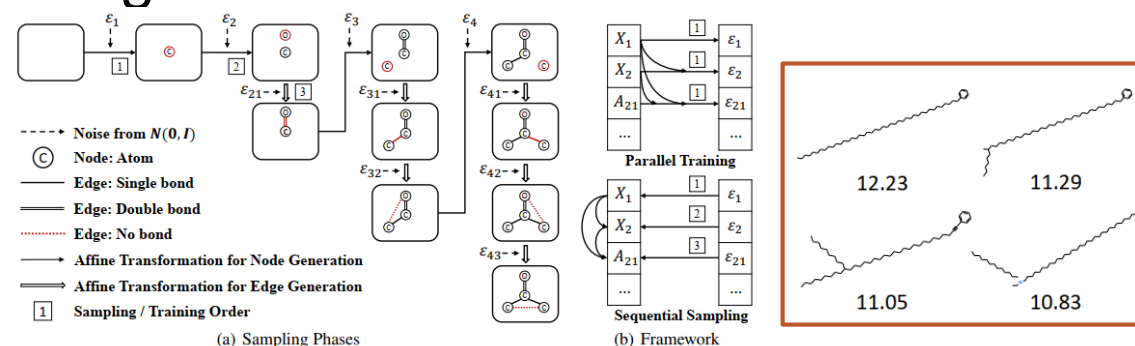
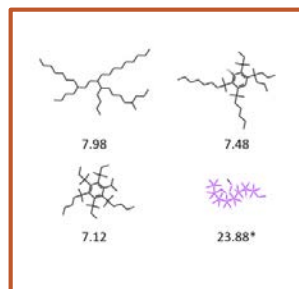
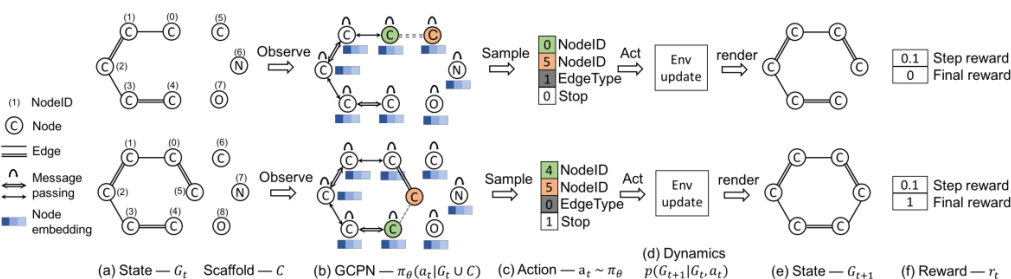


Image from: You et al. 2018. [Graph Convolutional Policy Network for Goal-Directed Molecular Graph Generation.](#) *NeurIPS*

Image from: Shi et al. 2020. [GraphAF: a Flow-based Autoregressive Model for Molecular Graph Generation.](#) *ICLR*

Related Works

□ Flow-based models

- GraphNVP: Graph Real-valued Non-Volume Preserving flow
 - ❖ Only use add coupling
- Limitations
 - ❖ Unstable deep structures, No chemical validity guarantee, Few novel molecules

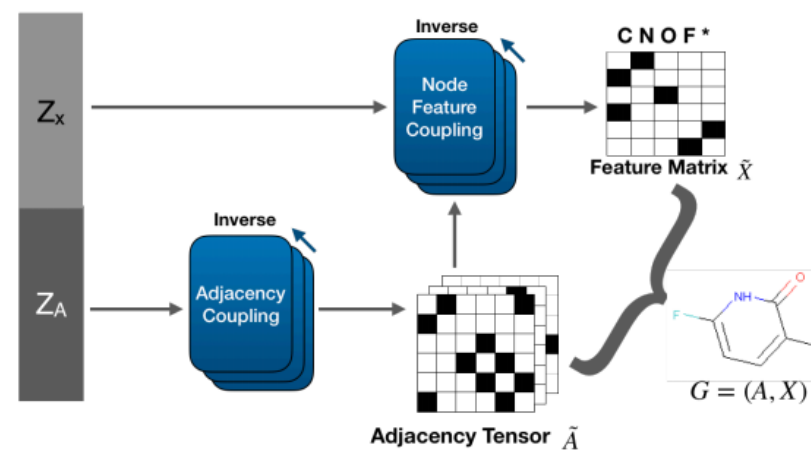
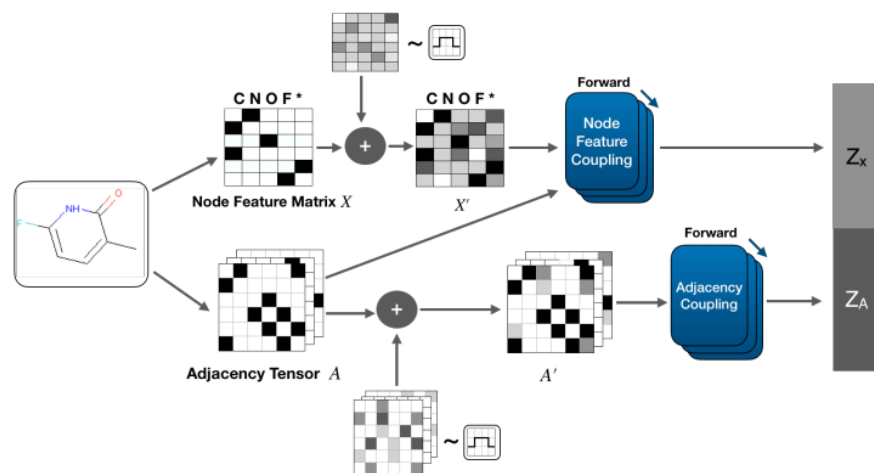


Image from: Madhawa et al. 2019. [GraphNVP: An Invertible Flow Model for Generating Molecular Graphs](#)

Related Works

❑ Classified by Data:

- Sequence: SMILES
- Graph: molecular graphs

❑ Classified by Deep Generative Models:

- Autoregressive Models (AR)
- Variational Autoencoders (VAE)
- Generative Adversarial Networks (GAN)
- Normalizing Flow Models (Flow)

❑ Classified by Search & Optimization

- Gradient ascend
- Reinforcement learning

Our Choice

❑ Classified by Data:

- Sequence: SMILES
- **Graph: molecular graphs**

❑ Classified by Deep Generative Models:

- Autoregressive Models (AR)
- Variational Autoencoders (VAE)
- Generative Adversarial Networks (GAN)
- **Normalizing Flow Models (Flow)**

❑ Classified by Search & Optimization

- **Gradient ascend**
- Reinforcement learning

Related works: Basics of Normalizing Flow

□ An invertible generative model

- Goal: $X \sim P(X)$

□ Inference: $Z = f_{\theta}(X)$

- From complex to simple, e.g. Z is Gaussian

□ Generation: $X = f_{\theta}^{-1}(Z)$

- Generate complex by invertible mapping

□ Exact Maximum Likelihood Training

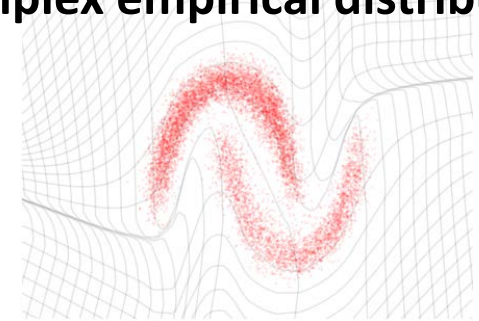
- Change of variable $\log P(X) = \log P(Z) + \log \left| \det \left(\frac{\partial f_{\theta}}{\partial Z} \right) \right|$

- $\operatorname{argmax}_{\theta} E_{M \sim P_{data}} [\log P_M(M; \theta)]$

□ Network structures:

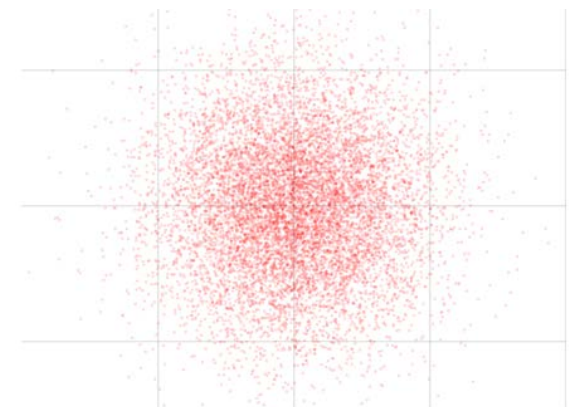
- f_{θ} : invertible DNNs, each layer is invertible
- Computing $\det \left(\frac{\partial f_{\theta}}{\partial Z} \right)$ should be efficient

$P(X)$:
Complex empirical distribution



Inference \Downarrow \Uparrow Generation
 $P(Z)$:

Simple latent distribution



Related works: NICE Model

- ❑ **NICE: Non-linear Independent Components Estimation**

- splitting dimensions + residual flow updated alternately

- ❑ **Split:**

- $X = (X_1, X_2)$
- $Z = (Z_1, Z_2)$

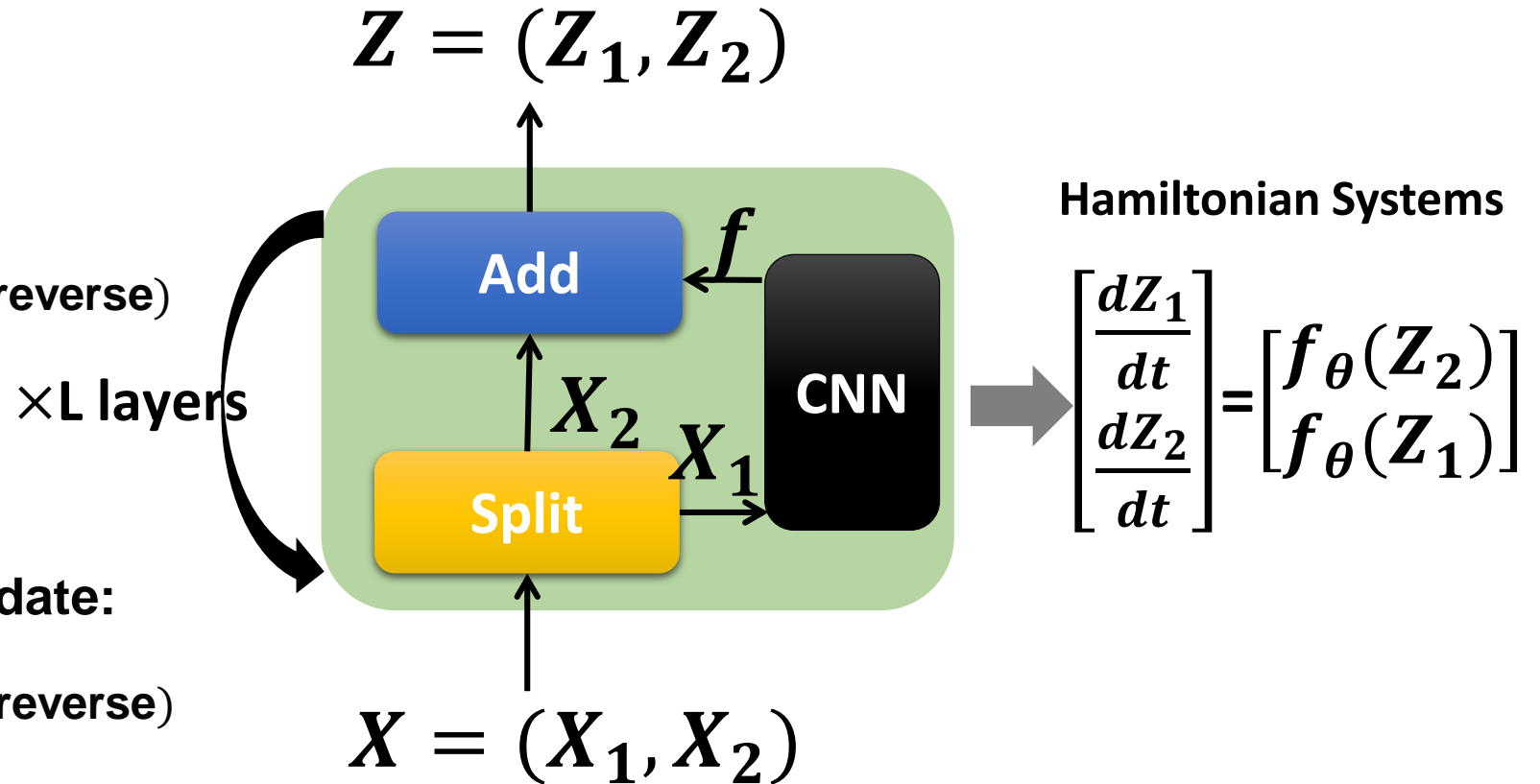
- ❑ **Add:**

- $Z_1 = X_1$ (save information for reverse)
- $Z_2 = X_2 + f_\theta(X_1)$ (Residual)
- Reverse mapping:
 - ❖ $X_1 = Z_1$
 - ❖ $X_2 = Z_2 - f_\theta(Z_1)$

- ❑ **Next layer by alternating update:**

- $Z_1 = X_1 + f_\theta(X_2)$ (Residual)
- $Z_2 = X_2$ (save information for reverse)

- ❑ ...



Dinh et al. 2014. [Nice: Non-linear independent components estimation](#)

Dinh et al. 2017. [Density Estimation using Real NVP](#). ICLR.

Chen et al. 2019. [Neural Ordinary Differential Equations](#). NeurIPS.

Related works: RealNVP Model

- RealNVP: Real-valued Non-Volume Preserving flow
 - splitting dimensions + affine updated alternately

- Split:**

- $X = (X_1, X_2)$
- $Z = (Z_1, Z_2)$

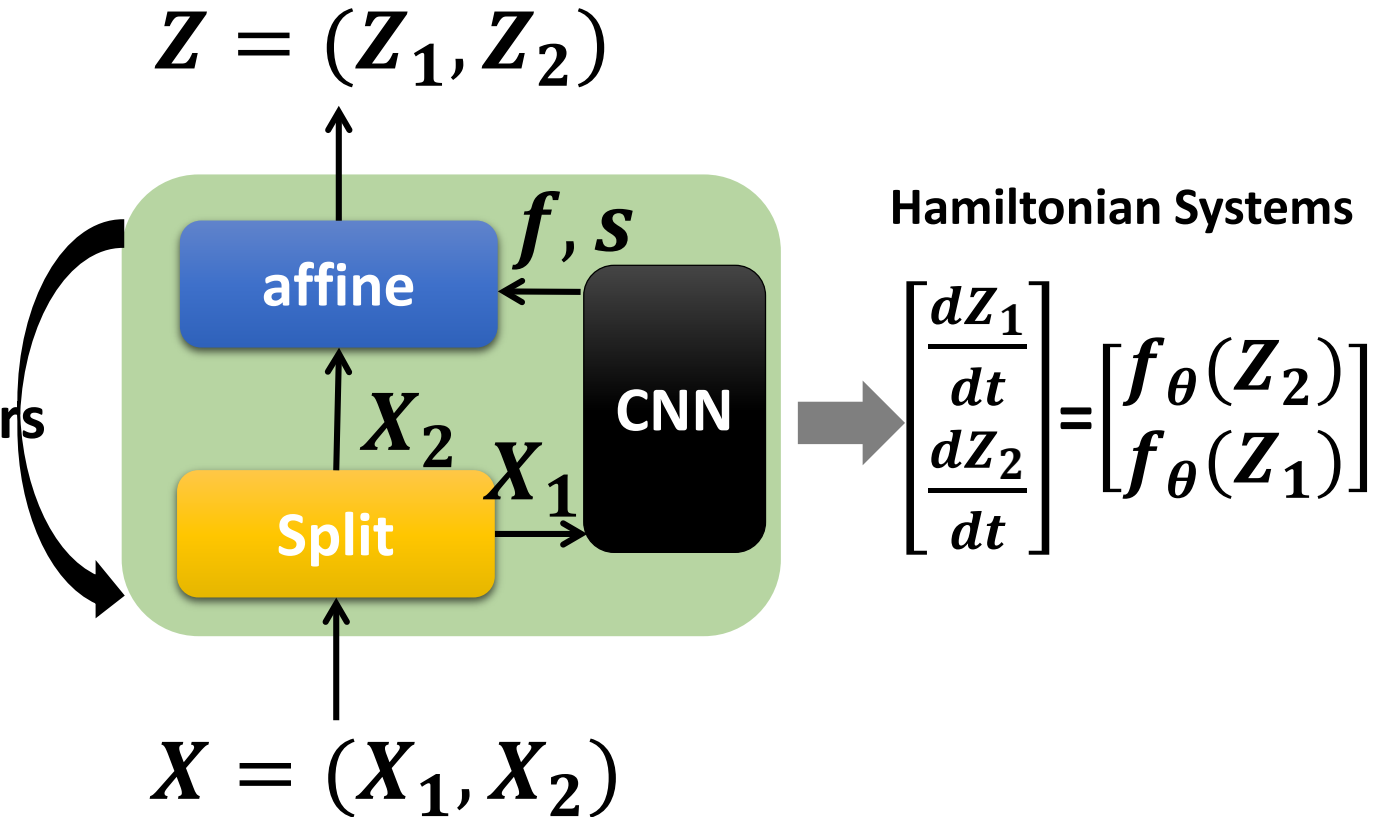
- Affine:**

- $Z_1 = X_1$ (save information for reverse)
- $Z_2 = X_2 e^{s\theta(X_1)} + f_\theta(X_1)$ (with scale)
- Reverse mapping:
 - $X_1 = Z_1$
 - $X_2 = e^{-s\theta(X_1)} [Z_2 - f_\theta(Z_1)]$

- Next layer by alternating update:**

- $Z_1 = X_1 e^{s\theta(X_2)} + f_\theta(X_2)$ (Residual)
- $Z_2 = X_2$ (save information for reverse)

- ...



Dinh et al. 2014. [Nice: Non-linear independent components estimation](#)

Dinh et al. 2017. [Density Estimation using Real NVP](#). ICLR.

Chen et al. 2019. [Neural Ordinary Differential Equations](#). NeurIPS.

Related works: Glow Model

□ **Glow: Generative flow with invertible 1*1 convolutions**

□ **Actnorm:**

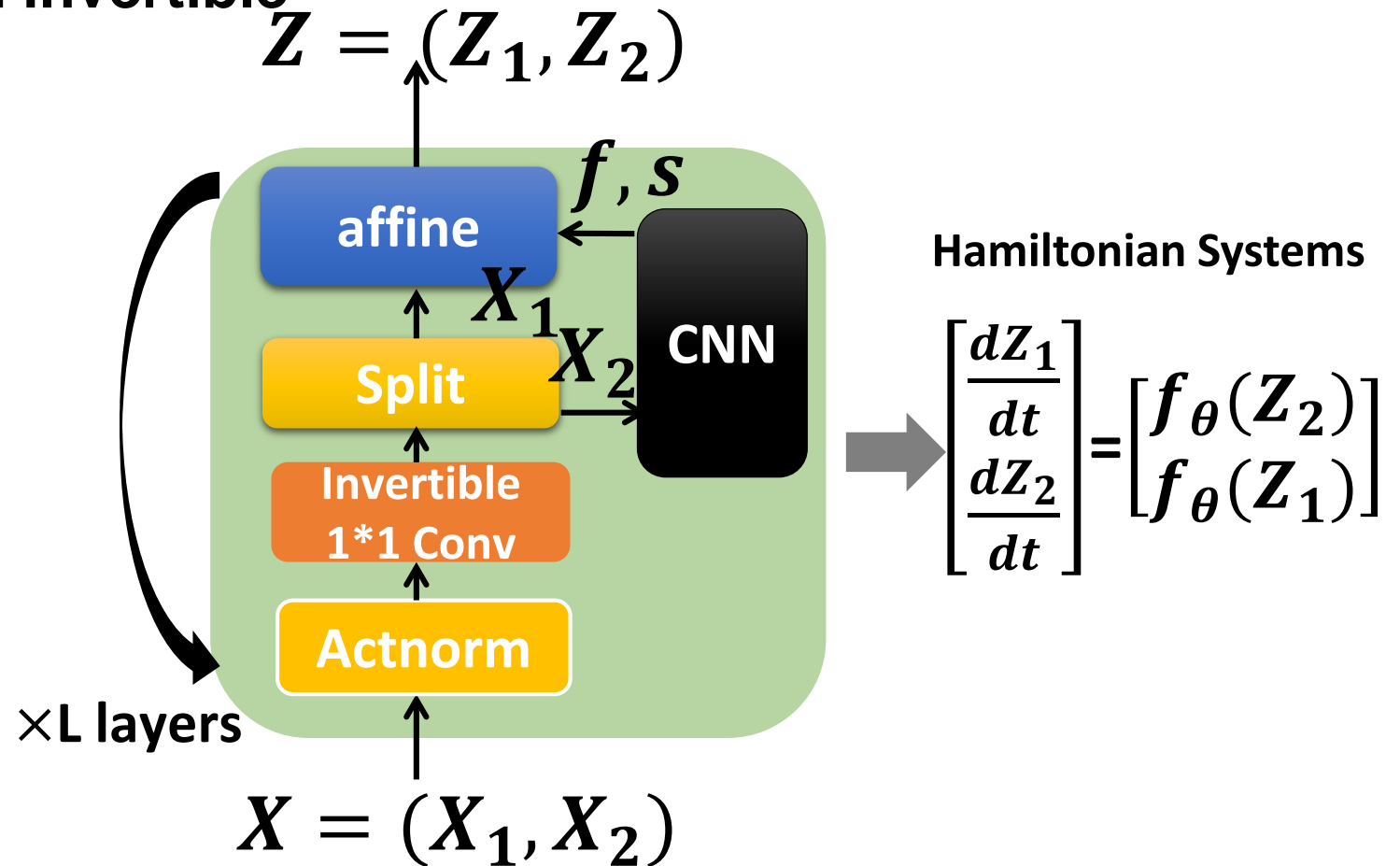
- Stable dynamics
- $B = \frac{B-\mu}{\sqrt{\sigma^2+\epsilon}}$ each channel over batch

□ **Invertible 1*1 convolution:**

- Expressive power
- $\mathbb{R}^{c \times n \times n} \times \mathbb{R}^{c \times c} \rightarrow \mathbb{R}^{c \times n \times n}$

□ **Affine:**

- $Z_1 = X_1$
- $Z_2 = X_2 e^{s\theta(X_1)} + f_\theta(X_1)$



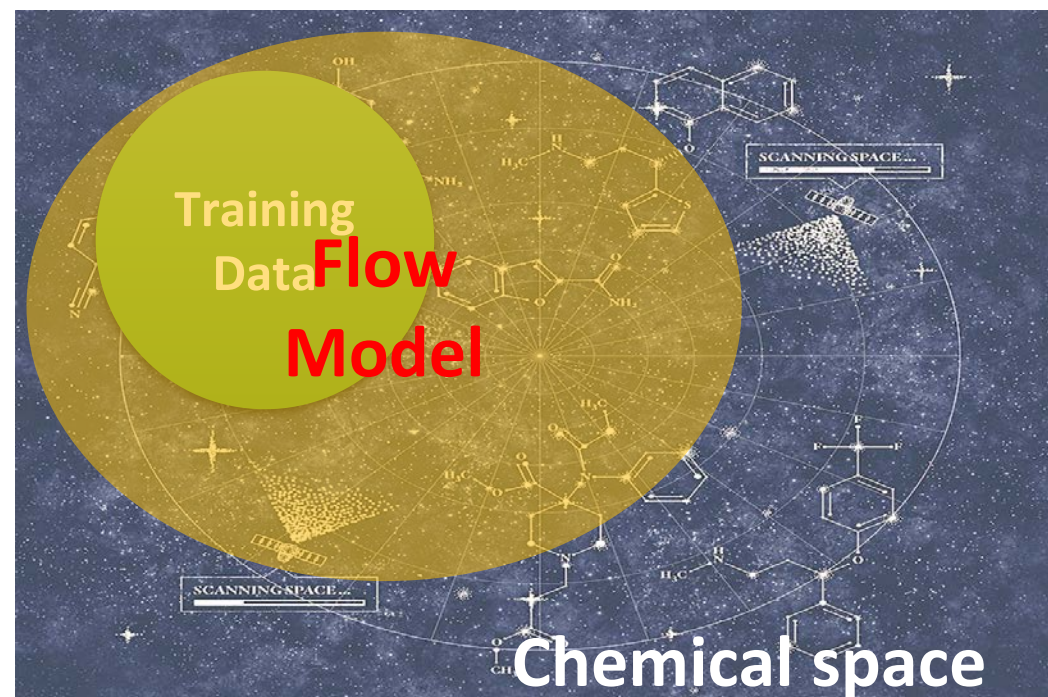
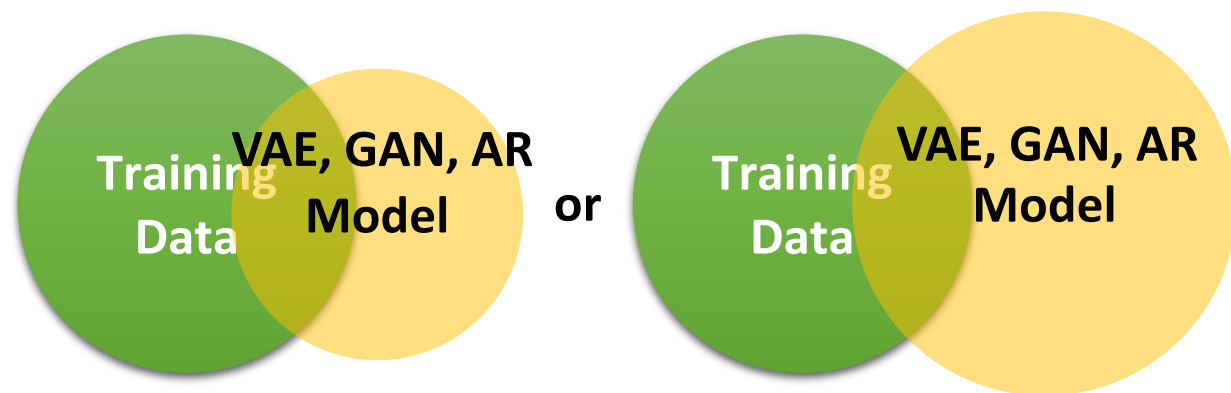
Chen et al. 2019. [Neural Ordinary Differential Equations](#). *NeurIPS*.

Kingma et al. 2018. [Glow: Generative flow with invertible 1x1 convolutions](#). *NeurIPS*.

Why Flow Frameworks

❑ Invertible mappings

- Potentials for generating more molecules
- VAE, GAN, AR are not invertible
- Flow learns a strict superset and represents chemical space better



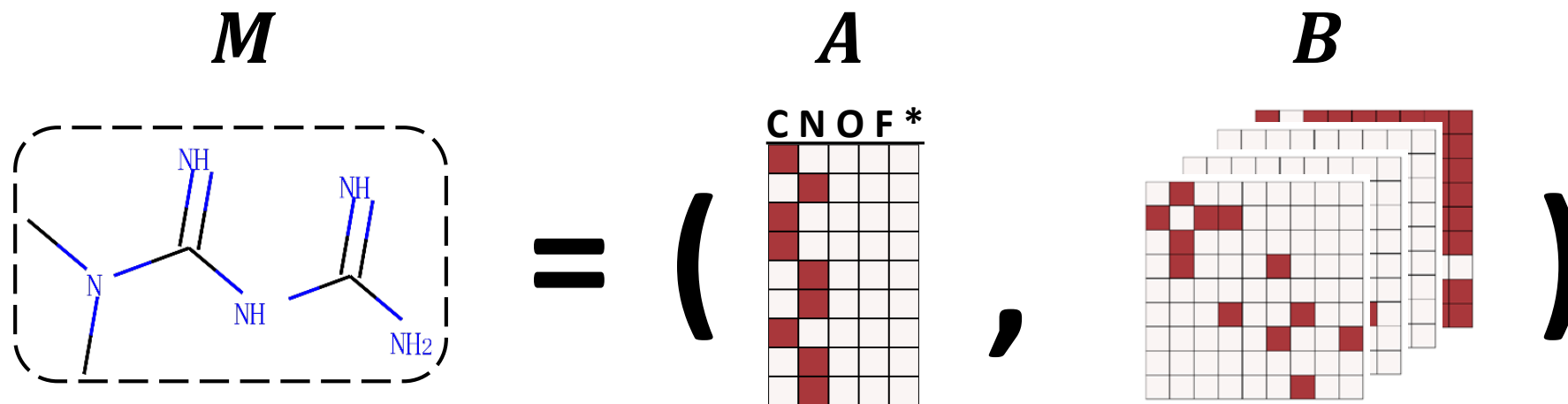
Why Flow Frameworks

- ❑ **Exact maximum likelihood training**
 - VAE, GAN are not
- ❑ **Efficient one-shot inference and generation**
 - Capturing molecular structures in a holistic way
 - AR is step-by-step
- ❑ **Better performance shown later**

Our MoFlow Model

□ Molecular Graph

- Molecules consist of atoms and bonds
- Molecule = (Atom, Bond)
- Atom $\in \{0,1\}^{n \times k}$, n Nodes in k (atom) types
- Bond $\in \{0,1\}^{c \times n \times n}$, Edges in c (bond) types

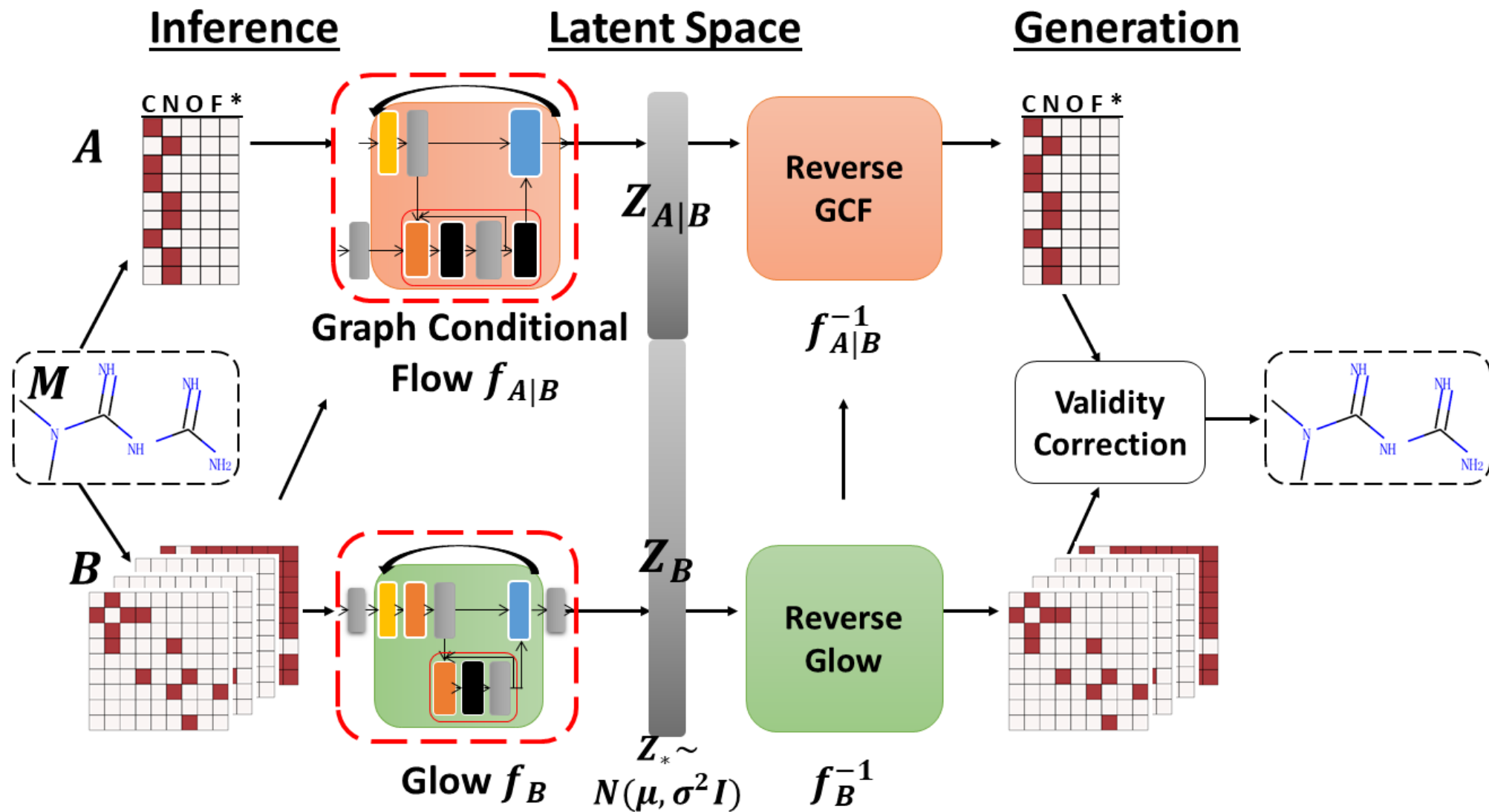


Our MoFlow Model

□ MoFlow:

- Molecule=(Atom, Bond) How to model intrinsic atom-bond structures of molecule?
- $P_M(M) = P_M((A, B)) \approx P_{A|B}(A|B)P_B(B)$
- Any flow model $f_B(B)$ for bonds $P_B(B)$
 - ❖ Generating graph skeleton by $P_B(B)$
- Graph conditional flow $f_{A|B}(A|B)$ for atoms given bonds $P_{A|B}(A|B)$
 - ❖ Generating nodes given graph skeleton by $P_{A|B}(A|B)$
- Assembling atom and bonds with validity correction

The Generative Framework



A variant of Glow for Bond

□ Squeeze

- $X \in \mathbb{R}^{c \times n \times n} \rightarrow \mathbb{R}^{ck^2 \times \frac{n}{k} \times \frac{n}{k}}$

□ Actnorm:

- Stable dynamics
- $B = \frac{B - \mu}{\sqrt{\sigma^2 + \epsilon}}$ each channel over batch

□ Invertible 1*1 convolution:

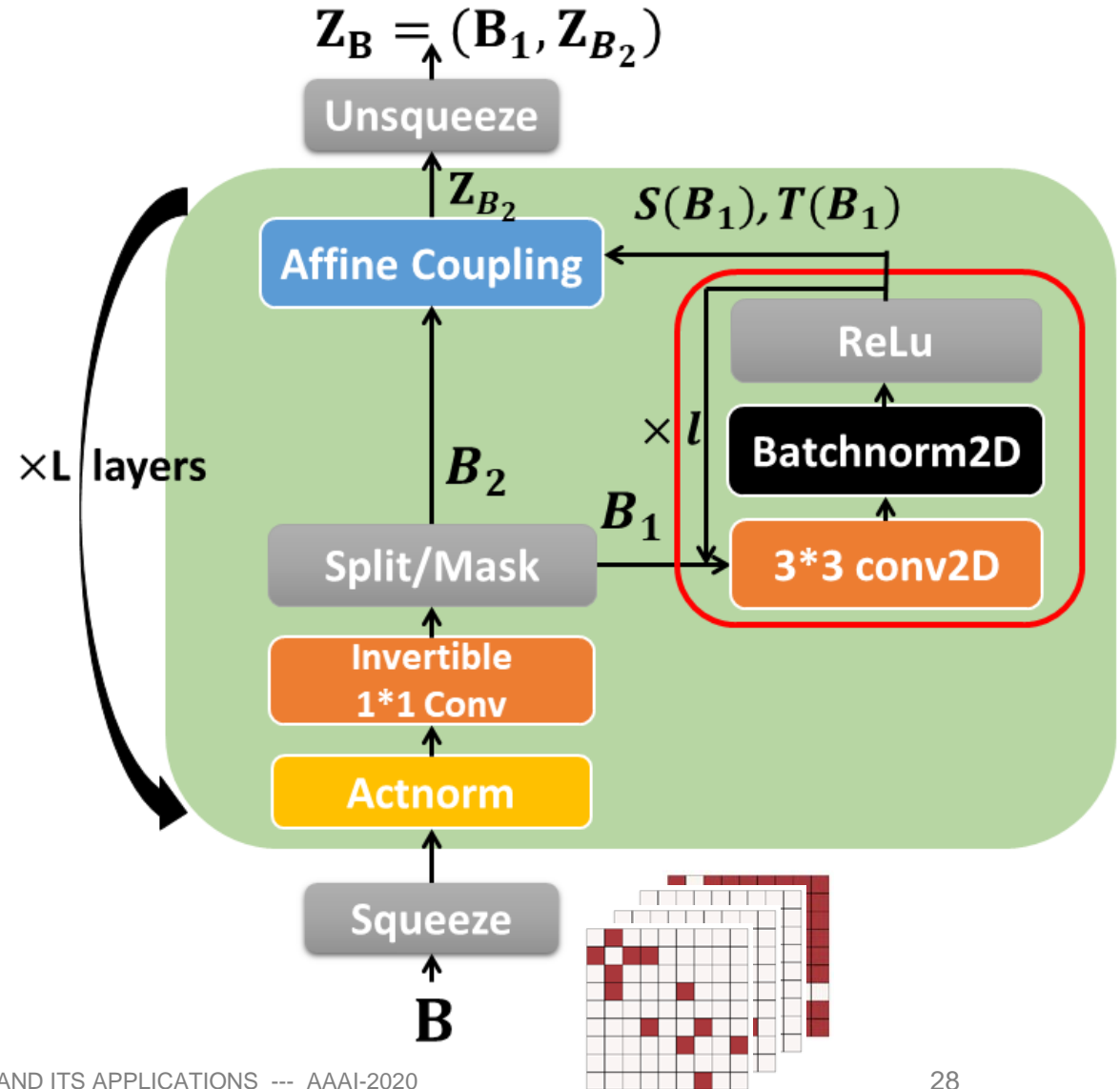
- Expressive power
- $\mathbb{R}^{c \times n \times n} \times \mathbb{R}^{c \times c} \rightarrow \mathbb{R}^{c \times n \times n}$

□ Split:

- Discretization of Hamiltonian system
- $\mathbf{B} = (\mathbf{B}_1, \mathbf{B}_2)$
- $\mathbf{Z} = (\mathbf{Z}_{B_1}, \mathbf{Z}_{B_2})$

□ Affine coupling:

- Stable (batchnorm2D, Sigmoid) and expressive power (Affine)
- $\mathbf{Z}_{B_1} = \mathbf{B}_1$
- $\mathbf{Z}_{B_2} = \mathbf{B}_2 \odot \text{Sigmoid}(S_\theta(\mathbf{B}_1)) + T_\theta(\mathbf{B}_1)$



Graph Conditional Flow For Atoms

Actnorm2D:

- Stable dynamics
- $B = \frac{B-\mu}{\sqrt{\sigma^2+\epsilon}}$ each row over batch

Split:

- Discretization of Hamiltonian system on Graphs
- $A = (A_1, A_2)$ by each row
- $Z = (Z_{A_1|B}, Z_{A_2|B})$

Graphnorm

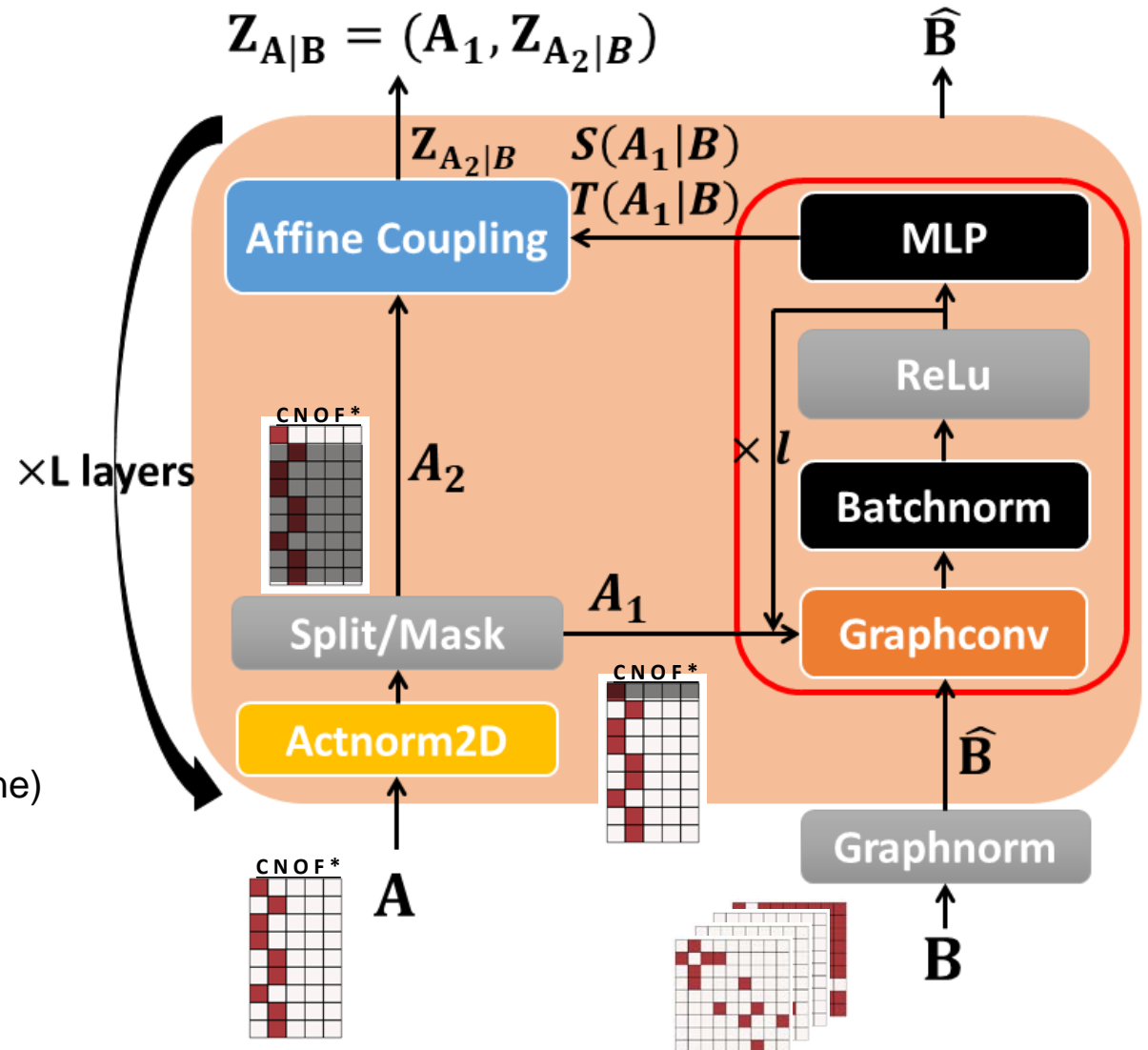
- $\hat{B}_i = D^{-1}B_i$, $D = \sum_{c,i} B_{c,i,j}$ in-degree over all channels

GraphConv(A|B)

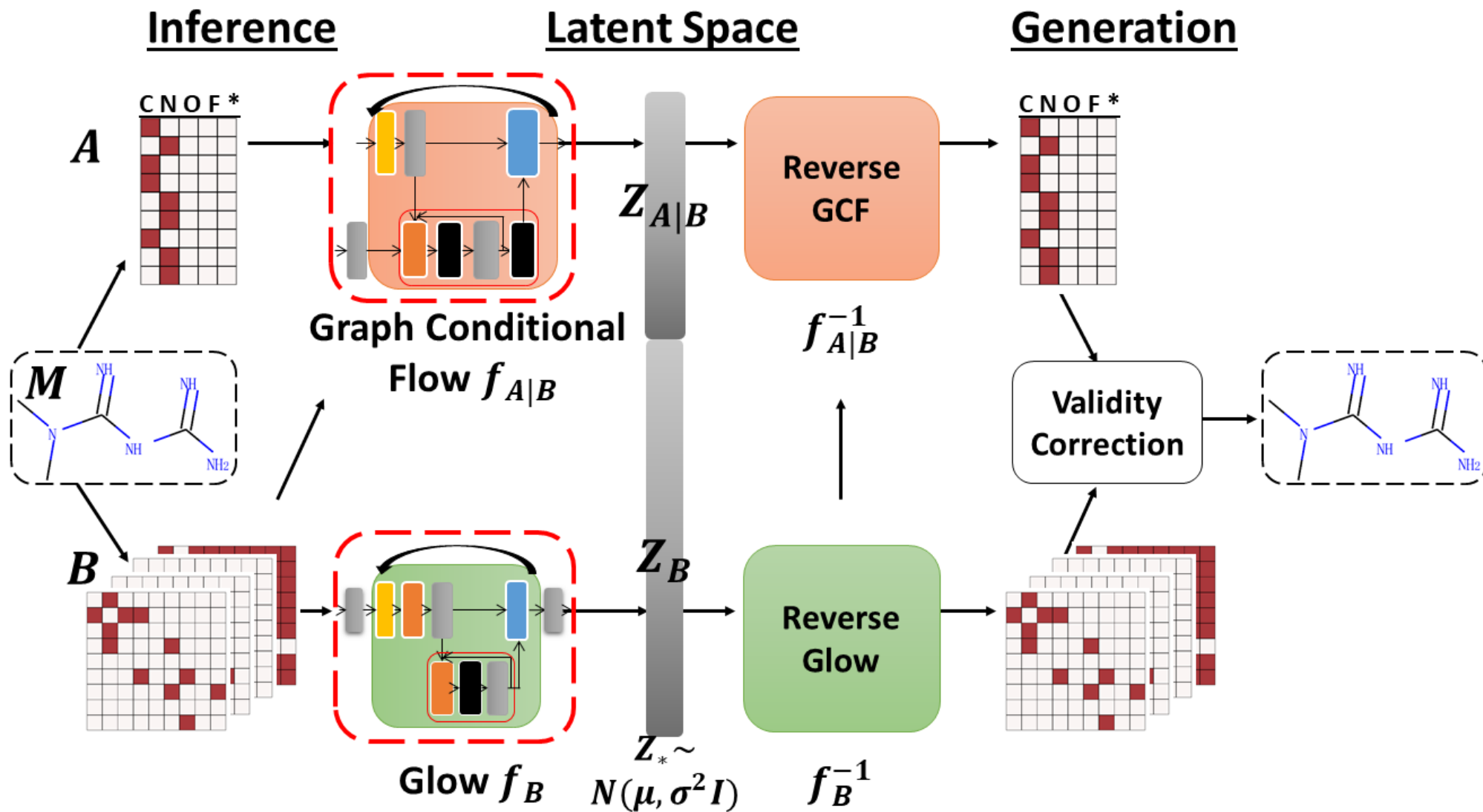
- $\sum_{i=1}^c \hat{B}_i (M \odot A) W_i + (M \odot A) W_0$
- update each row by the remaining rows

Affine coupling:

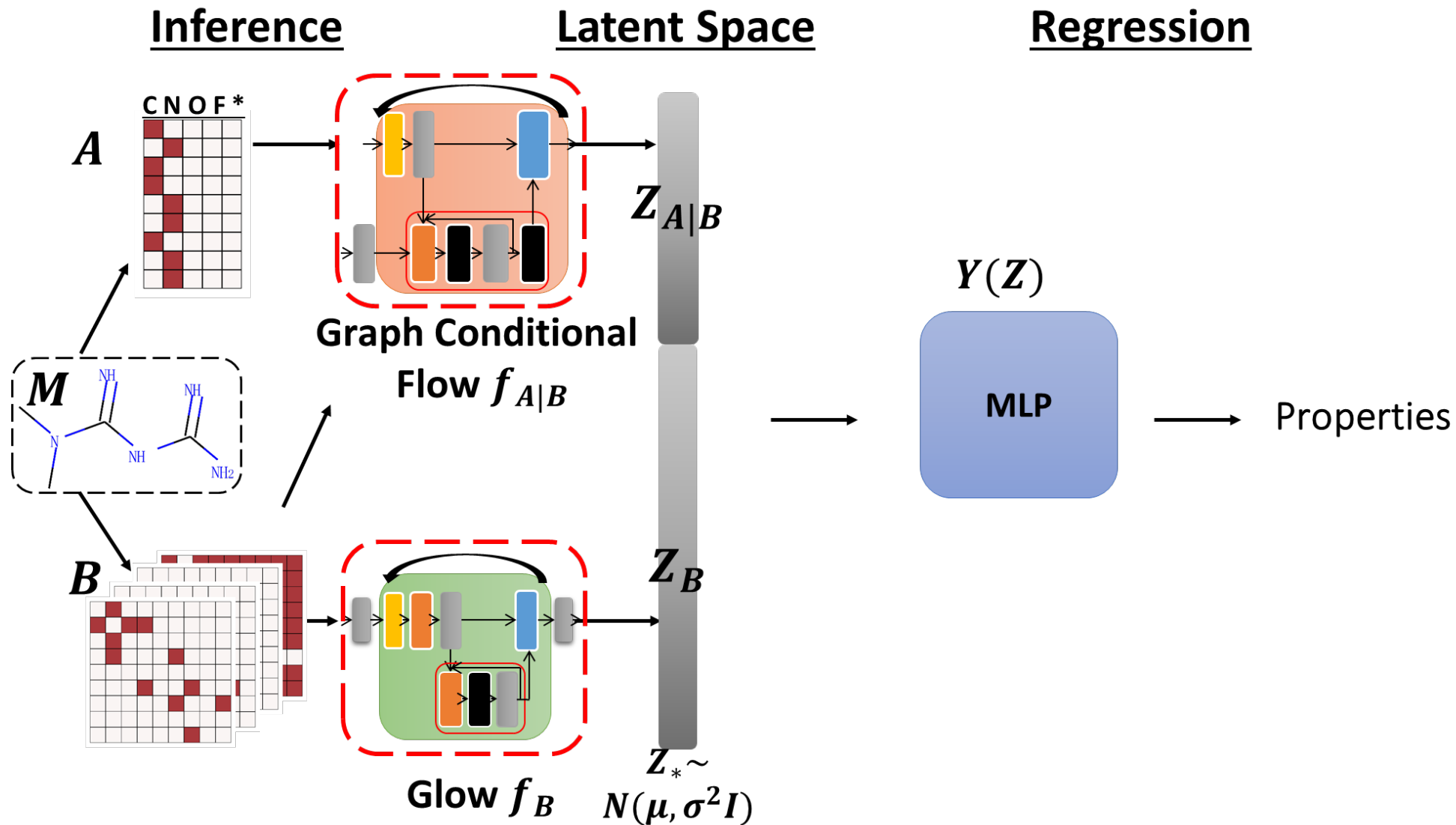
- Stable (batchnorm, Sigmoid) and expressive power (Affine)
- $Z_{A_1|B} = A_1$
- $Z_{A_2|B} = A_2 \odot \text{Sigmoid}(S_\theta(A_1|B)) + T_\theta(A_1|B)$



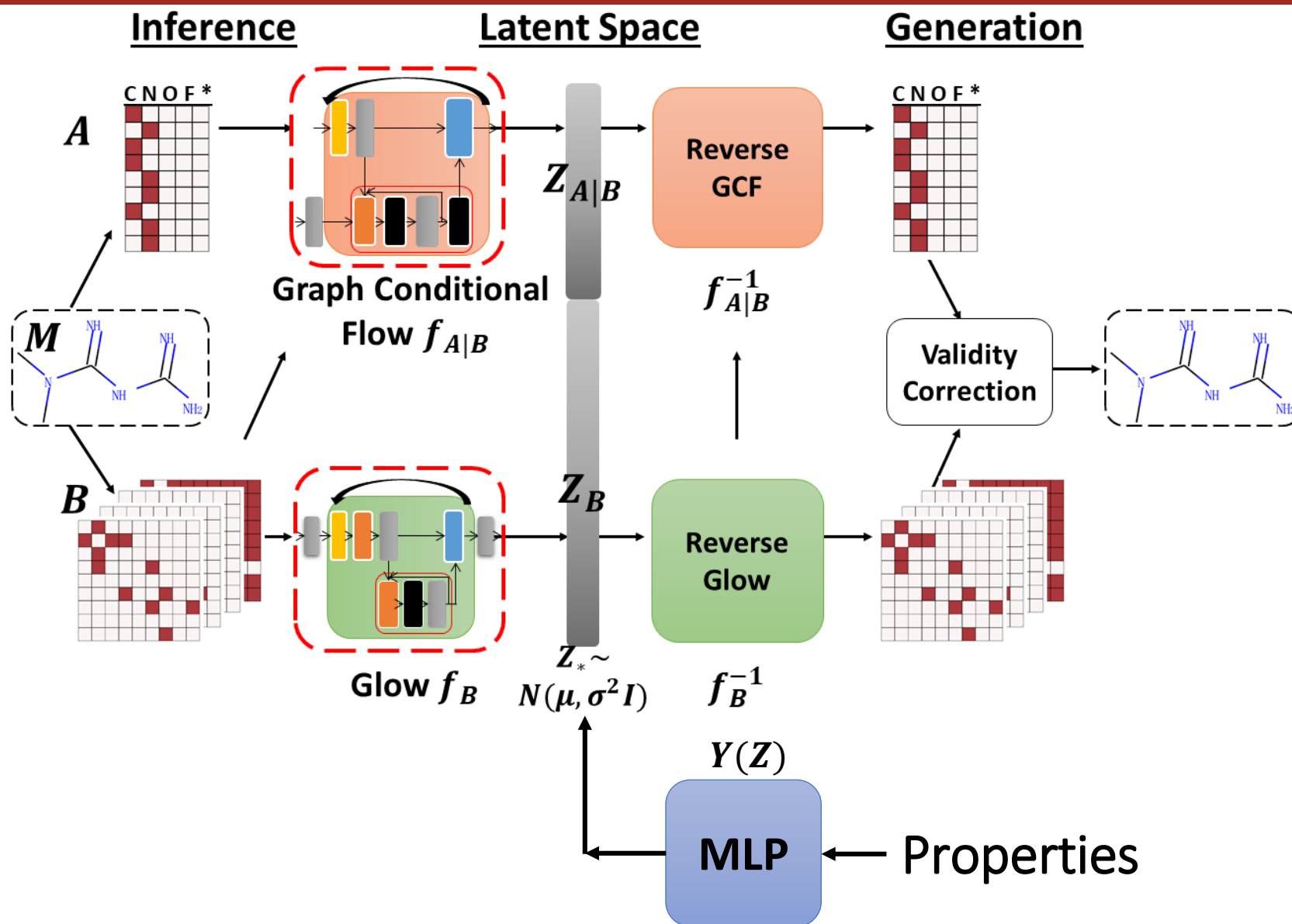
Molecular Graph Generation



Graph Property Prediction



Molecular Graph Optimization



Validity Correction

□ Valid molecules: valency constraints

- $\sum_{c,j} B(c, i, j) \leq \text{Valency}(\text{Atom}_i) + \text{Formal_Charge}$
- C: 4, O:2, O+:3

□ Valid Correction

- While checking valency constraints:
 - ❖ If follows constraints:
 - Return the greatest connected component
 - ❖ Else:
 - Delete unnecessary bond or add charge to atoms according to rules

Experiments

- 1. Molecular Generation & Reconstruction**
- 2. Visualization of Continuous Latent Space**
- 3. Property Optimization**
- 4. Constrained Property Optimization**

EXP1: Molecular Generation & Reconstruction

□ The Problem:

- Input: $\{M_1, M_2, \dots\}$ molecules
- Model
 - ❖ P_M Learned molecular generative model
 - ❖ Generation: $M = f^{-1}(Z)$, Z follows isotropic Gaussian
 - ❖ Reconstruction: $Z = f(M)$ and $M = f^{-1}(Z)$
- **Goal:** To generate valid & unique & novel molecules

□ Datasets:

○

	#Graphs	#Nodes	#Node/Atom Types	#Edge/Bond Types
QM9	134K	9	4	3
ZINC	250K	38	9	3

EXP1: Molecular Generation & Reconstruction

□ Evaluation metrics:

1. **Validity**: %chemically valid molecules in all the generated molecules
2. **Validity without check/correction**
3. **Uniqueness**: %chemically valid and unique molecules in all the generated molecules
4. **Novelty**: %generated valid molecules not in training dataset
5. **Reconstruction rate**: % training dataset which can be reconstructed from their latent representations
6. **N.U.V.**: %novel, unique and valid molecules in all the generated molecules

EXP1: Molecular Generation & Reconstruction

- More novel & unique & valid molecules
 - than previous models
- 100% Reconstruction
 - Strict superset of training dataset
- Better validity without check
 - Than AR models. One-shot models, a holistic way
- Our MoFlow represents and explores the chemical space better!

Table 1: Generative performance on QM9

	% Validity	% Validity w/o check	% Uniqueness	% Novelty	% N.U.V.	% Reconstruct
GraphNVP	83.1 ± 0.5	-	99.2 ± 0.3	58.2 ± 1.9	47.97	100
GRF	84.5 ± 0.70	-	66.0 ± 1.15	58.6 ± 0.82	32.68	100
GraphAF	100	67	94.51	88.83	83.95	100
MoFlow	100.00 ± 0.00	95.74 ± 0.65	99.48 ± 0.33	98.69 ± 0.39	98.18 ± 0.53	100.00 ± 0.00

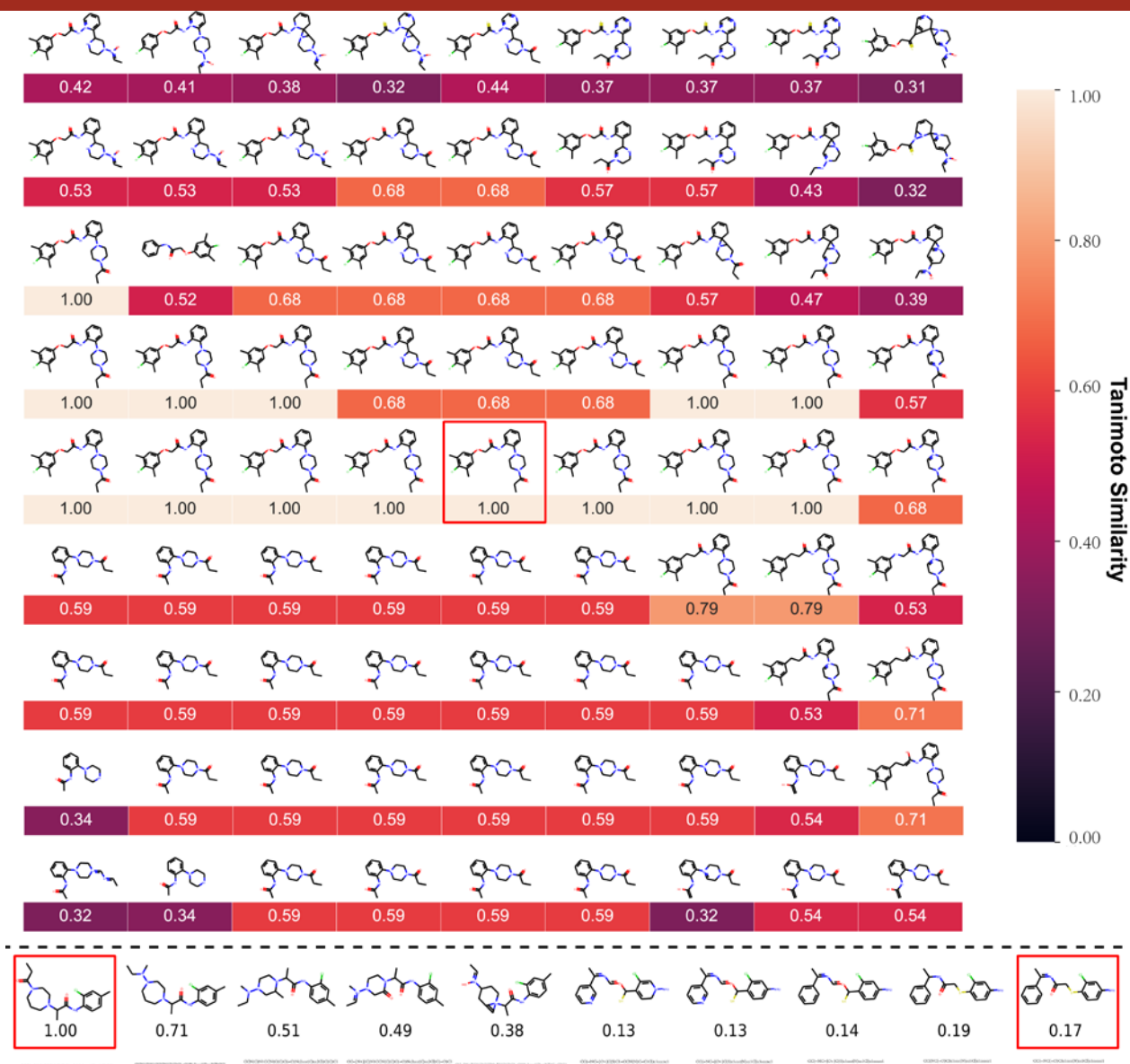
Table 2: Generative performance on Zinc250k

	% Validity	% Validity w/o check	% Uniqueness	% Novelty	% N.U.V.	% Reconstruct
JT-VAE	100	-	100	100	100	76.7
GCPN	100	20	99.97	100	99.97	-
MRNN	100	65	99.89	100	99.89	-
GraphNVP	42.6 ± 1.6	-	94.8 ± 0.6	100	40.38	100
GRF	73.4 ± 0.62	-	53.7 ± 2.13	100	39.42	100
GraphAF	100	68	99.10	100	99.10	100
MoFlow	100.00 ± 0.00	81.94 ± 0.45	99.94 ± 0.05	100.00 ± 0.00	99.94 ± 0.05	100.00 ± 0.00

EXP2: Visualization of latent space

Embedding molecular graphs into continuous latent space

- Grid interpolation around one seed molecule
 - ❖ Smooth latent space \leftrightarrow Similar chemical structures (Tanimoto similarity)
- Linear interpolation between two molecules
 - ❖ Changing trajectory from one to another



EXP3: Property Optimization

❑ To Generate Novel Molecules with the best Quantitative Estimate of Druglikeness (QED) scores as many as possible

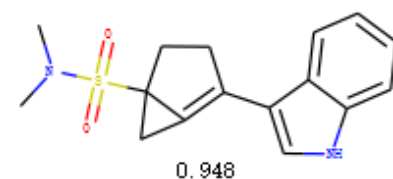
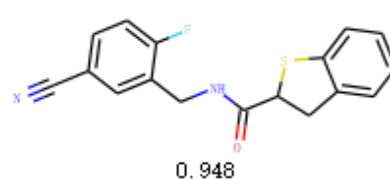
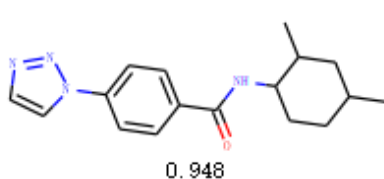
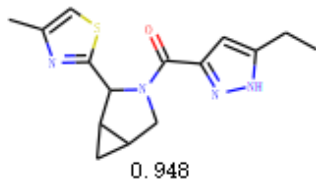
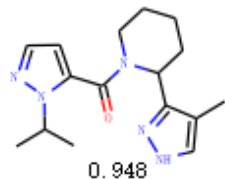
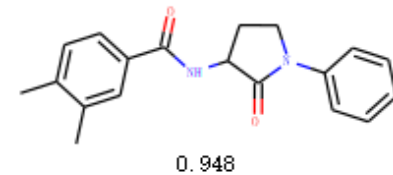
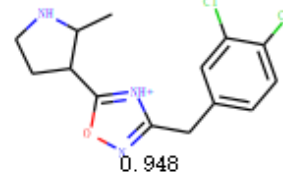
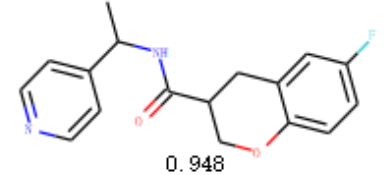
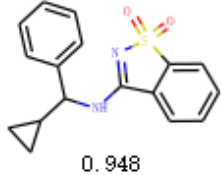
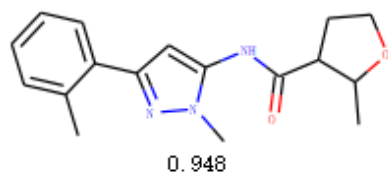
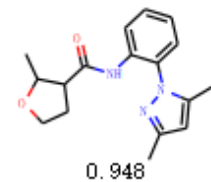
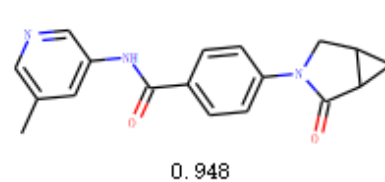
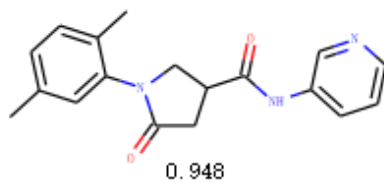
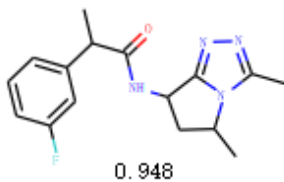
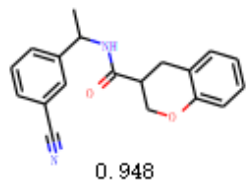
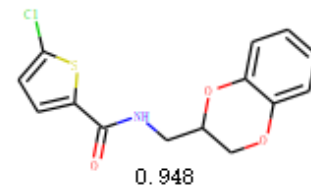
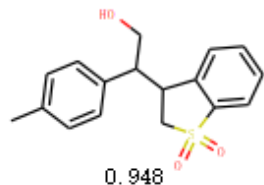
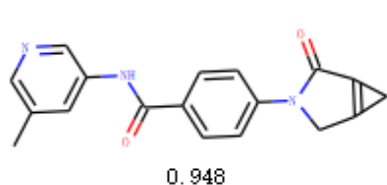
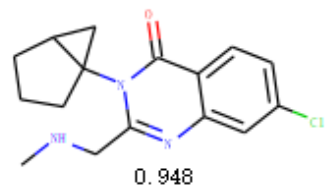
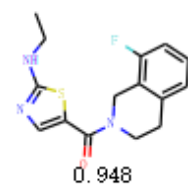
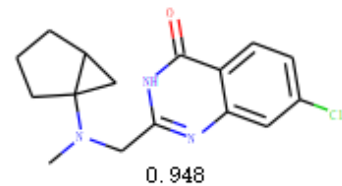
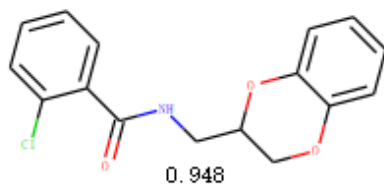
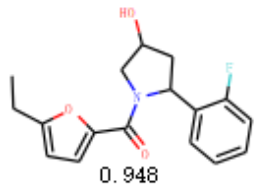
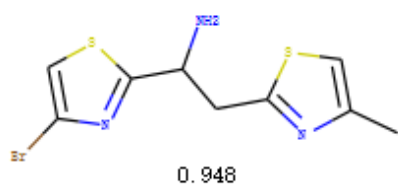
○ Searching latent space by gradient ascend

❑ Much more molecules with top QED scores

Table 3: Discovered novel molecules with top QED score. Our MoFlow finds more molecules with the best QED score. More results in

Method	1st	2nd	3rd	4th
ZINC (Dataset)	0.948	0.948	0.948	0.948
JT-VAE	0.925	0.911	0.910	-
GCPN	0.948	0.947	0.946	-
MRNN	0.948	0.948	0.947	-
GraphAF	0.948	0.948	0.947	0.946
MoFlow	0.948	0.948	0.98	0.948

EXP3: Property Optimization



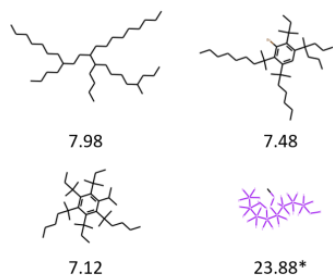
EXP4: Constrained Property Optimization

□ Find a new molecule M' from a seed molecule M

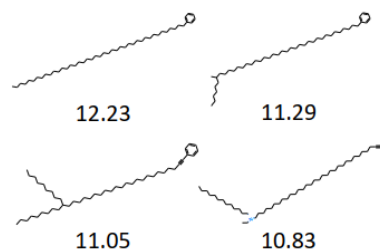
- To maximize: $\text{Similarity}(M, M')$ and $Y(M') - Y(M)$
 - ❖ Tanimoto similarity of Morgan fingerprint
 - ❖ Target property Y : penalized $\log P$ (plogP), the octanol-water partition coefficients ($\log P$) penalized by the synthetic accessibility (SA) score and number of long cycles.

EXP4: Constrained Property Optimization

- ❑ Best similarity
- ❑ Second best improvement
- ❑ AR+RL model tends to generate long chains



GCPN

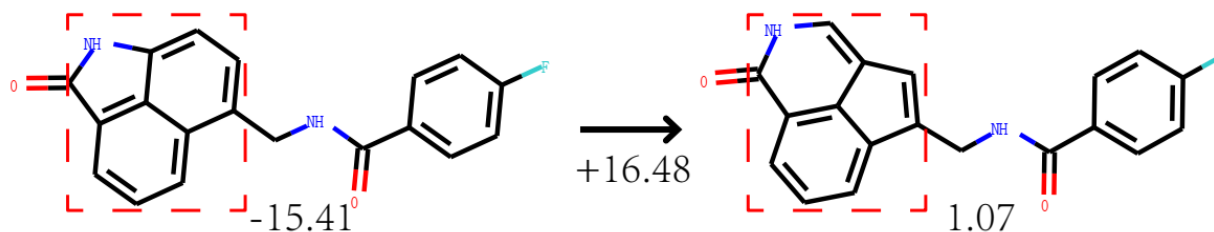


GraphAF

Table 4: Constrained optimization on Penalized-logP

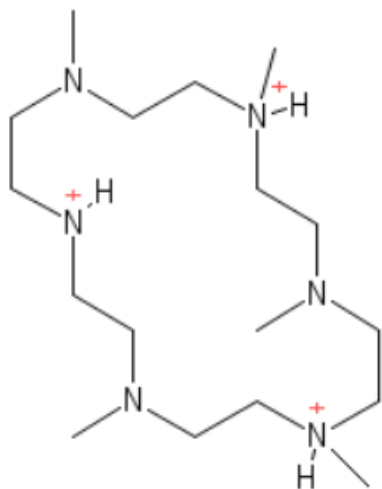
δ	JT-VAE			GCPN		
	Improvement	Similarity	Success	Improvement	Similarity	Success
0.0	1.91 ± 2.04	0.28 ± 0.15	97.5%	4.20 ± 1.28	0.32 ± 0.12	100%
0.2	1.68 ± 1.85	0.33 ± 0.13	97.1%	4.12 ± 1.19	0.34 ± 0.11	100%
0.4	0.84 ± 1.45	0.51 ± 0.10	83.6%	2.49 ± 1.30	0.48 ± 0.08	100%
0.6	0.21 ± 0.71	0.69 ± 0.06	46.4%	0.79 ± 0.63	0.68 ± 0.08	100%

δ	GraphAF			MoFlow		
	Improvement	Similarity	Success	Improvement	Similarity	Success
0.0	13.13 ± 6.89	0.29 ± 0.15	100%	8.61 ± 5.44	0.30 ± 0.20	98.88%
0.2	11.90 ± 6.86	0.33 ± 0.12	100%	7.06 ± 5.04	0.43 ± 0.20	96.75%
0.4	8.21 ± 6.51	0.49 ± 0.09	99.88%	4.71 ± 4.55	0.61 ± 0.18	85.75%
0.6	4.98 ± 6.49	0.66 ± 0.05	96.88%	2.10 ± 2.86	0.79 ± 0.14	58.25%



EXP4: Constrained Property Optimization

CN1CC[NH+](C)CCN(C)CC[NH+](C)CCN(C)CC[NH+](C)CC1

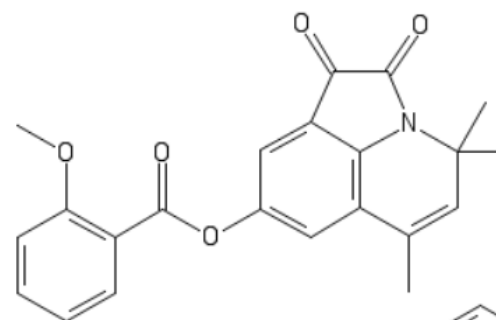


PlogP: -49.7182

+47.87

PlogP: -1.849

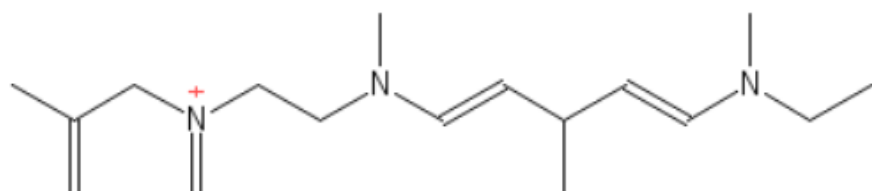
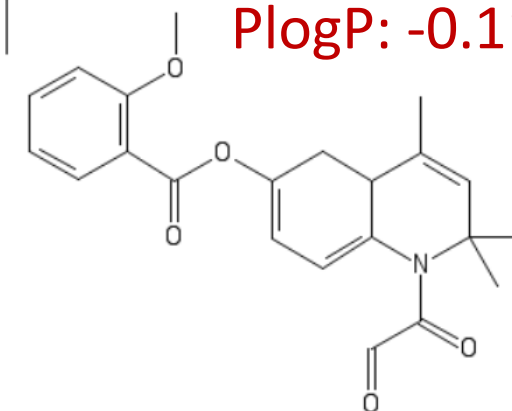
COc1ccccc1C(=O)Oc1cc2c3c(c1)C(C)=CC(C)(C)N3C(=O)C2=O



PlogP: -15.962

+15.85

PlogP: -0.11607



C=C(C)C[N+](=C)CCN(C)C=CC(C)C=CN(C)CC

COc1ccccc1C(=O)OC1=CC=C2C(C1)C(C)=CC(C)(C)N2C(=O)C=O

Summary

□ Novel MoFlow model

- A variant of Glow for bonds
- Novel Graph conditional flow for atoms given bonds
- Novel validity correction
- Invertible, fast inference and generation at one shot

□ The state-of-the-art results

- Best results for generation and reconstruction
 - ❖ w.r.t. novelty, uniqueness, validity, and reconstruction rate
- Best results for QED property optimization
 - ❖ More drug-like molecules
- Best similarity scores for constraint optimization and second best improvement scores for plogP

This Tutorial

- ✓ **Molecular Graph Generation:** to generate novel molecules with optimized properties
 - Graph generation
 - Graph property prediction
 - Graph optimization
- **Learning Dynamics on Graphs:** to predict temporal change or final states of complex systems
 - Continuous-time network dynamics prediction
 - Structured sequence prediction
 - Node classification/regression
- **Mechanism Discovery:** to find dynamical laws of complex systems
 - Density Estimation vs. Mechanism Discovery
 - Data-driven discovery of differential equations



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Part 1: MoFlow: An Invertible Flow Model for Generating Molecular Graphs

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Weill Cornell Medicine

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Thank You!



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Differential Deep Learning on Graphs and its Applications

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