



**Weill Cornell  
Medicine**

# Differential Deep Learning on Graphs and its Applications

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# This Tutorial

- ❑ [www.calvinzang.com/DDLG\\_AAAI\\_2020.html](http://www.calvinzang.com/DDLG_AAAI_2020.html)
- ❑ [AAAI-2020](#)
- ❑ **Friday, February 7, 2020, 2:00 PM -6:00 PM**
- ❑ **Sutton North, Hilton New York Midtown, NYC**



# This Tutorial

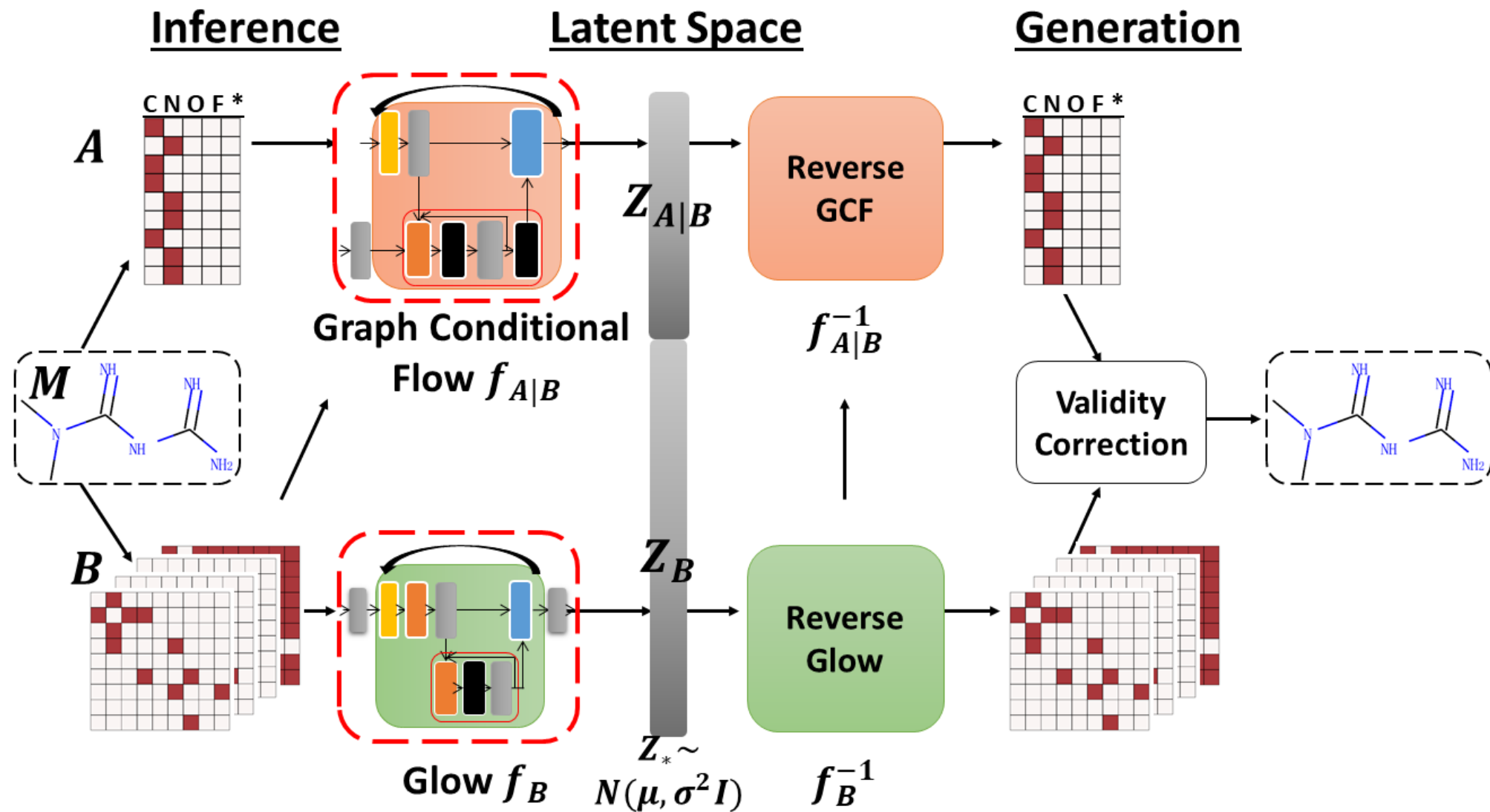
- ✓ **Molecular Graph Generation:** to generate novel molecules with optimized properties
  - Graph generation
  - Graph property prediction
  - Graph optimization
- ✓ **Learning Dynamics on Graphs:** to predict temporal change or final states of complex systems
  - Continuous-time network dynamics prediction
  - Structured sequence prediction
  - Node classification/regression
- ✓ **Mechanism discovery:** to find dynamical laws of complex systems
  - Density Estimation vs. Mechanism Discovery
  - Data-driven discovery of differential equations

# Molecular Graph Generation

- **Goal:** To generate novel molecules with optimized properties
- **Graph Analysis tasks**
  - Graph generation:  $G \sim P(G)$
  - Graph property prediction:  $f(G)$
  - Graph optimization:  $G \rightarrow G'$  and maximizing  $f(G') - f(G)$



# MoFlow: An Invertible Flow Model for Generating Molecular Graphs



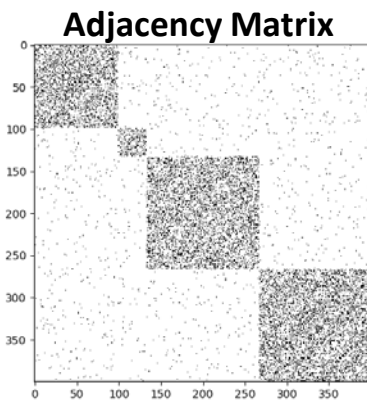
# Learning Dynamics on Graphs

□ **Goal:** To predict temporal change or final states of complex systems

□ **Graph Analysis tasks**

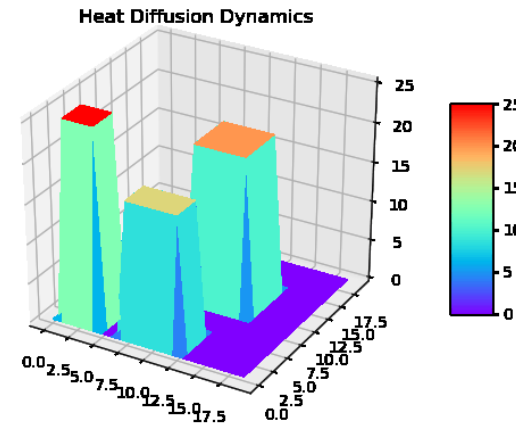
- Continuous-time network dynamics prediction  $X(t)$
- Structured sequence prediction  $X[t + 1]$
- Node classification/regression  $Y(X(T))$

Graph



+

Dynamic Process



Dynamics of each nodes



# Neural Dynamics on Complex Networks

## Our Model :

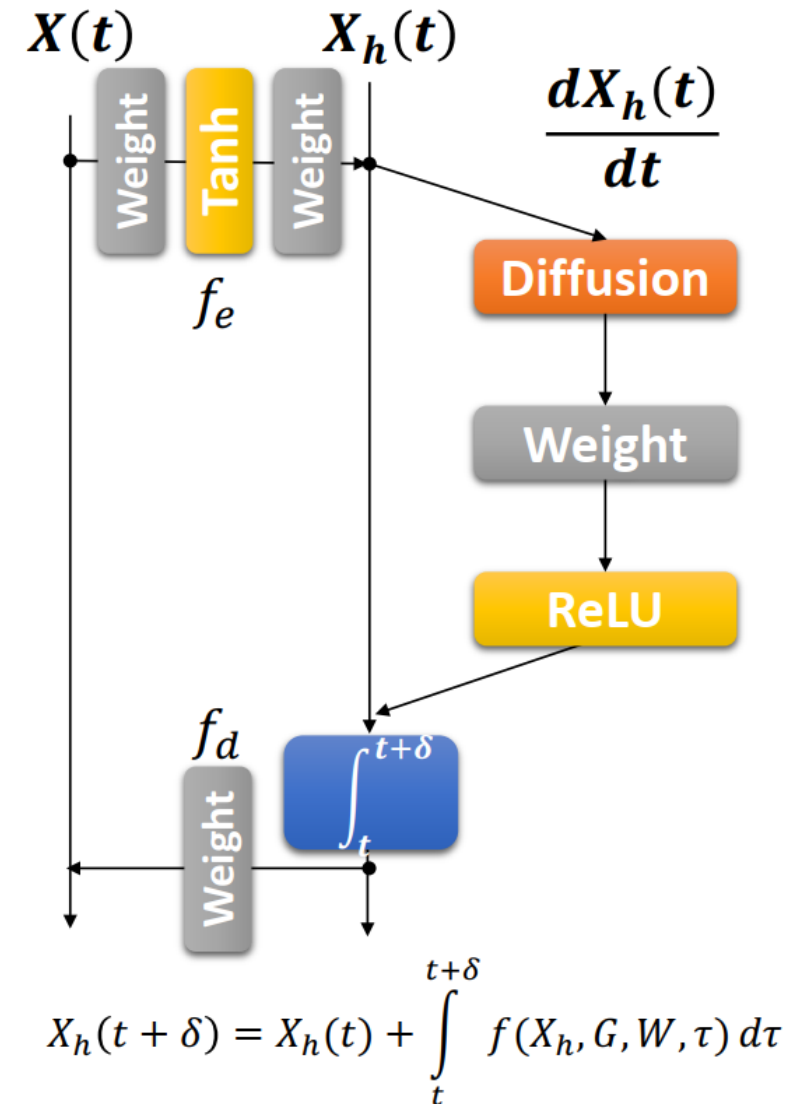
$$\operatorname{argmin}_{W_*, b_*} \mathcal{L} = \int_0^T |X(t) - \hat{X}(t)| dt$$

$$\text{subject to } X_h(t) = \tanh(X(t)W_e + b_e)W_0 + b_0$$

$$\frac{dX_h(t)}{dt} = \text{ReLU}(\Phi X_h(t)W + b), X_h(0)$$

$$X(t) = X_h(t)W_d + b_d$$

$$\Phi = D^{-\frac{1}{2}}(D - A)D^{-\frac{1}{2}} \in \mathbb{R}^{n \times n}$$

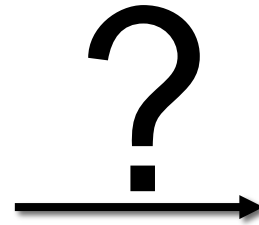
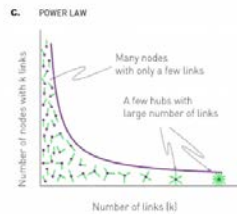
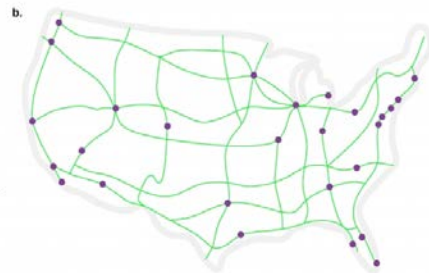
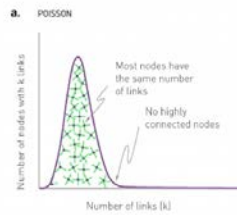


# Mechanism Discovery

□ **Goals:** To find dynamical laws of complex systems

□ **Graph Analysis tasks**

- Density estimation vs. mechanism discovery
- Data-driven discovery of differential equations



$$\frac{dX}{dt}$$

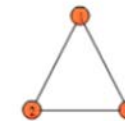
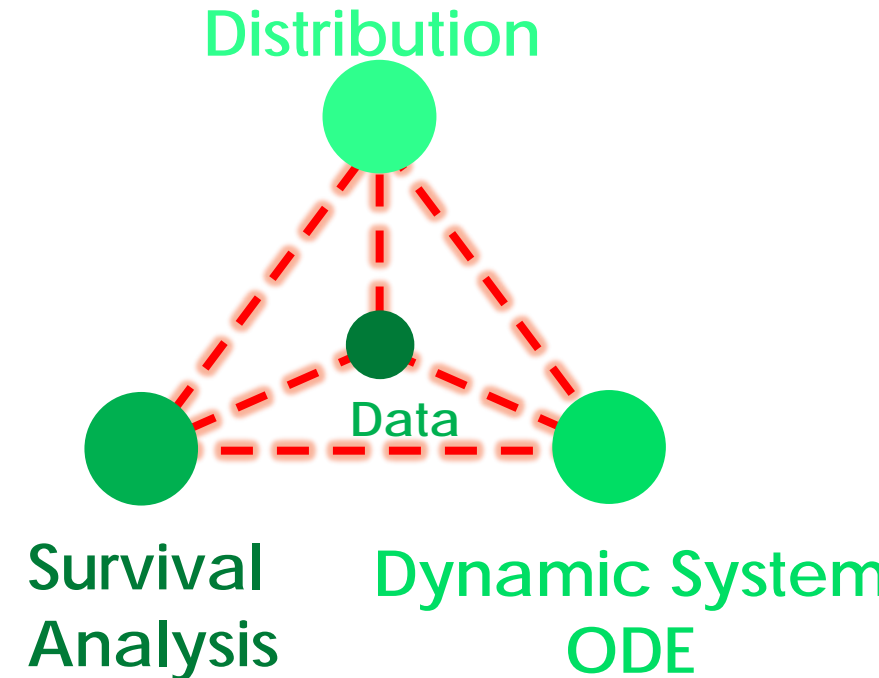


Image from <http://networksciencebook.com/chapter/4#hubs>



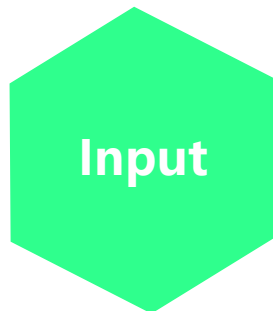
# Dynamical Origins of Distribution Functions

- A theorem constructing dynamic systems described by Differential Equations which generate the observed distribution

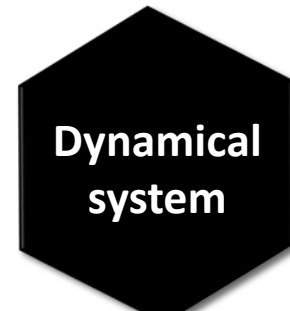


Forward: data generation

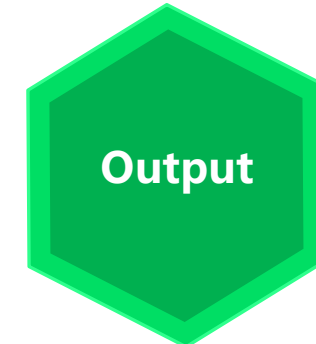
Stochastic, linear



Deterministic, nonlinear



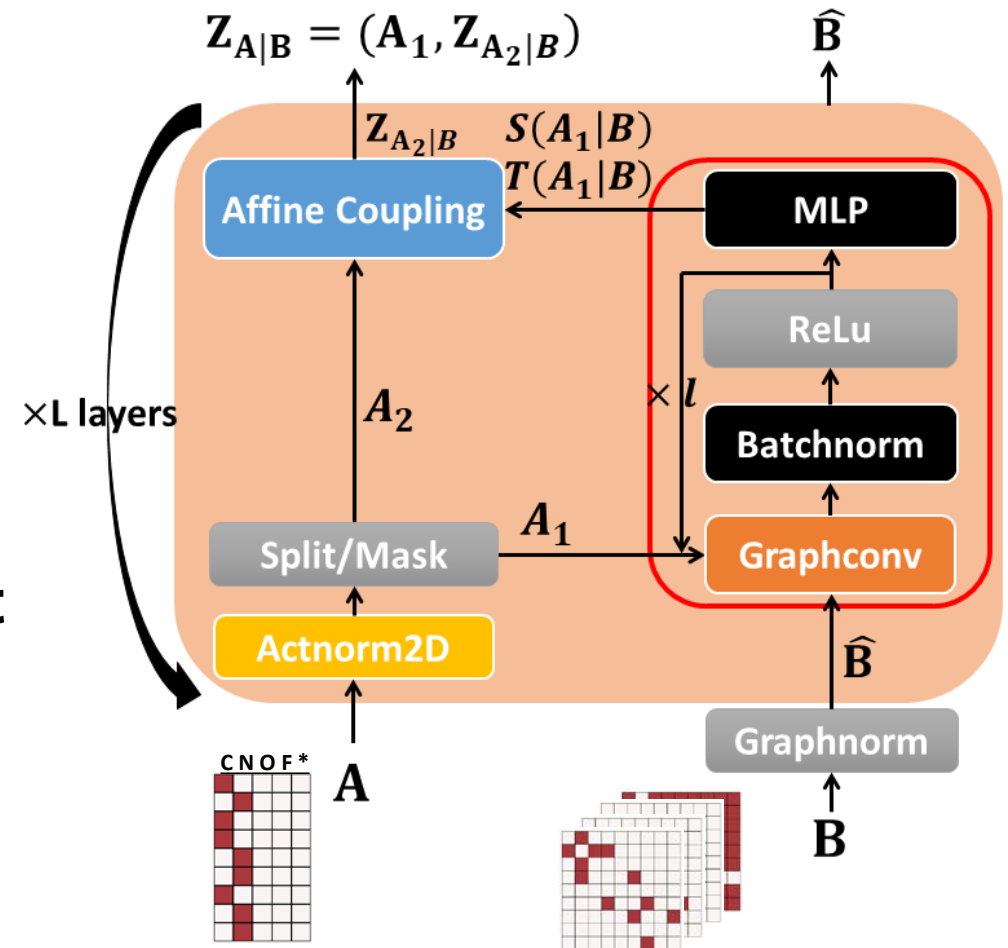
Complex Pattern



Backward:  
system  
identification/Le  
arning

# Some Practical Tips

- ❑ **Data preprocessing**
  - Padding null atoms, augmenting null edges
- ❑ **Normalization matters**
  - Graphnorm, batchnorm, actnorm
- ❑ **Stable flows with less reconstruction error**
  - Normalization, sigmoid, checking each layer
- ❑ **Discrete mapping is faster than integration**
- ❑ **Split and coupling layer are very efficient invertible framework for graph convolution**
- ❑ **Visualizing dynamics on graphs**
- ❑ **Thinking physical meanings of differential equations**



# Differential Deep Learning on Graphs

- ❑ **Graphs and Differential Equations are general tools to describe structures and dynamics of complex systems**
- ❑ **Inspired by the Differential Equations, we can design and analyze Deep Models**
- ❑ **For applications on graphs (our focus), including:**
  - Molecular Graph Generation
  - Learning dynamics of complex systems
  - Mechanism discovery

**in a data-driven manner**

# More Directions

## □ Deep Learning → Differential Equations

### ○ Analysis

- ❖ Math analysis tools
- ❖ Concepts in dynamic system and control: stability, robustness, complexity, resilience, etc.
- Modeling Continuous-time process
  - ❖ Physical meaning. The laws of nature are expressed as differential equations.

## □ Differential Equations → Deep Learning

### ○ Design

- ❖ There are many dynamical systems and differential equations.
- ❖ Discretization of continuous time-varying neural dynamics → Deep Neural Networks
- ❖ DNNs implemented by modern auto-differentiation softwares are more flexible, expressive and efficient
- Generative models and Invertible structures

# More Directions

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## □ Applications

- Network medicine
- **Drug discovery**
- Molecular dynamics
- Urban computing
- **Social networks**
- Recommendation
- Etc. (structures + dynamics)

# Thank You!



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